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« Contents »

New Format for *The Mathematics Teacher* ..... *W. D. Reeve* 5
Research in Mathematics Education ..... *W. D. Reeve* 6
Different Kinds of Equality ..... *C. C. McDuffee* 10
A Redefinition of Secondary School Mathematics ... *Leroy H. Schnell* 14
Recreational Aspects of Mathematics in the Junior High School .... *Anna R. Meeks* 20
Activity in Mathematics—The Slow-Moving Pupil ..... *Virgil S. Mallory* 23
Travel Motivates Mathematics ..... *Eoline Wallace Moore* 27
The Future of Geometry ..... *Margaret Joseph* 29
An Attempt at Solving the Problem of Individual Differences..... *Clara D. Murphy* 32
The Art of Teaching 39
Abstracts of Recent Articles on Mathematical Topics in Other Periodicals ..... *Nathan Lazar* 40
News Notes 45
New Books 49

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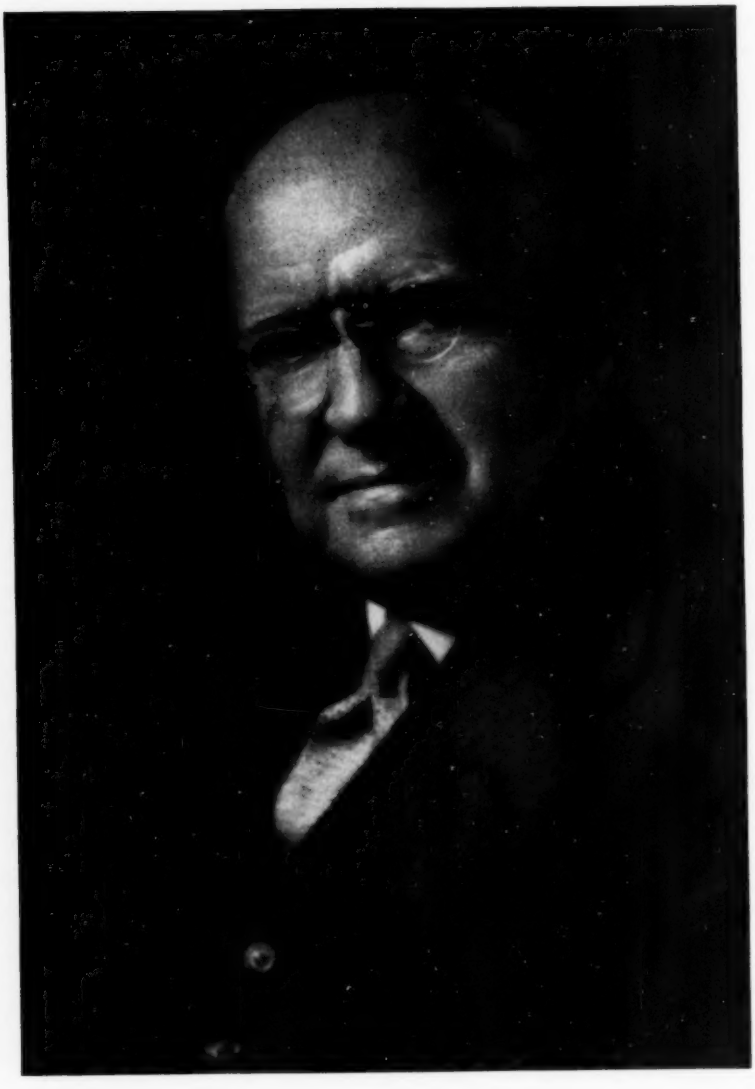

David Eugene Smith

The Mathematics Teacher takes great pleasure in presenting this portrait of Professor Smith in anticipation of his 76th birthday which will occur on January 21, 1936.

THE MATHEMATICS TEACHER

Volume XXIX Number 1

Edited by William David Reeve

New Format for the Mathematics Teacher

By W. D. REEVE, *Editor*

WITH this issue, *The Mathematics Teacher* is changed in form and in size. The new format enables us to give our readers more material and to present it in a more desirable way. This is done by increasing the size of the page so as to permit two columns of printed matter to appear. This double column effect makes the reading process easier and the artistic effect better. Along with this change in format, it is hoped that we shall be able to introduce other new features that will make the magazine more attractive and helpful to teachers of mathematics. What we can ultimately do to improve the magazine will depend largely upon the financial support the National Council of Teachers of Mathematics can get from those in whose interest the magazine is published eight times each year. With renewed efforts on the part of our present membership we should be able to secure many new subscribers. No meeting of mathematics teachers should be held without someone's taking the responsibility of presenting the case for the *Mathematics Teacher*. Subscription blanks for use at such meetings like the one appearing as the first advertisement of this issue can be had postpaid for the asking. The blanks also give a description of the National Council Yearbooks which now constitute an essential part of the library of many teachers of mathematics. We shall be glad to have the comments of our readers on the new format and also any suggestions for the further improvement of the magazine along any particular line.

HIGHER MATHEMATICS

The high wheel of heaven turns
With ten million shining urns,
A transcendent arc of light
In the interstellar night.

Flaming suns with fiery veils
Balance in their burning scales,
Spin their curves, and shining come
By a radiant rule of thumb.

Starry patterns, starry laws,
Buried in the primal cause,
Marshall to the cold decree
Of a bright geometry.

The mathematics of the sky
Is frightening, when such as I
Cannot with impunity
Resolve the simple rule of three.

—WILFRED J. FUNK

Research in Mathematics Education*

By W. D. Reeve
Teachers College, Columbia University

Since I have been asked to speak on the need of, and opportunity for research in mathematics education, I will confine my remarks to what I think may be done along research lines to solve some of the problems of a modern mathematics curriculum.

In building a curriculum in mathematics as in any other subject four major steps are necessary:

I. Selecting the objectives to be attained.
II. Choosing the content material best adapted to the realization of the objectives.
III. Developing adequate methods of teaching and learning.
IV. Setting up a complete and efficient testing program.

Obviously any curriculum building program must be a dynamic affair so as to give ample opportunity for the training of teachers in service. And since it must be going on all the time any curriculum building program that is adequate will always afford opportunity for research of one kind or another.

I will, therefore, refer only to a few types of research which are most commonly employed in educational circles and which may be employed in the four steps referred to above.

Research has been classified on the basis of the manner in which the problem in question is attacked. We might still be helped considerably by a further refinement of our classification and by a more intelligent statement of what research actually is. However, that cannot be taken up in the short time at my disposal.

Types of Research

Some of the most frequently used types of research are:

1. Fact Finding Research.

The value of this type of work and its limitations were set forth by Dr. Briggs[1] so well in an earlier conference of this series that I will not discuss it here.

Several of our recent studies in mathematics here at Teachers College have made use of certain relevant facts the methods of gathering which are now generally known and would not be considered as research technique by some although they have done much more than gather facts. A recent study by Sueltz on "The Status of Teachers of Secondary Mathematics" is an example of such a study as is also an earlier study by McCormick on "The Teaching of General Mathematics in the Secondary Schools of the United States," where besides the gathering of certain facts, important analyses, interpretations, and recommendations were made.

2. Historical Research.

Anyone interested in the historical approach to the solution of some problem in the teaching of mathematics should consult such studies as that of Sanford on "The History and Significance of Certain Standard Problems in Algebra" or Walker on "Studies in the History of Statistical Method."

This type of research has also been discussed at length by Dr. Briggs in his recent paper, but an example of where it might be used in mathematics may be given here.

It is often thought by some teachers of mathematics that the movement to introduce the study of informal geometry into the junior high school is an innovation. The fact is that the introduction of informal geometry is a practice which good teachers have always observed. A good historical study would reveal the facts. Betz in his chapter on "The Teaching of Intuitive Geometry" in the Eighth Year-

[1] Dr. Briggs' paper on "Research in Secondary Education" can be secured from the Bureau of Publications, Teachers College, Columbia University.

* One of a series of lectures given at Teachers College, Columbia University last year at a Conference on Frontier Research in Problems of Instruction.

book of the National Council of Teachers of Mathematics has made an excellent start on such a study.

3. Statistical Research.

This type of research has already been carefully discussed in an earlier conference and will not be elaborated here. It ought to be said, however, that this type of research has been overdone. Some educators have been inclined to give their approval grudgingly to other types of research who will unquestionably favor anything that savors of statistics. Such people are always insisting on objective evidence in spite of the fact that innate wisdom is supposed to be a desirable trait.

Knowledge dwells in heads replete with thoughts of other men,
Wisdom in minds attentive to their own.
Knowledge a rude unprofitable mass,
The mere materials with which wisdom builds,
Till smoothed and squared and fitted to its place,
Does but encumber whom it seems to enrich.
Knowledge is proud that he knows so much;
Wisdom is humble that he knows no more.

COWPER

Again it is true that although after certain careful statistical studies have been carried on at a great expenditure of time and effort, important findings have been made, and certain procedures have been recommended, nothing is done by workers in the field to make such studies the basis of improving either the materials or the methods of instruction. It is needless to say that "half-baked" statistical studies are practically worthless. Either we should make the results of our statistical studies more understandable or educate our teachers as to their worth and importance in classroom practice.

4. Experimental Research.

This is undoubtedly a very important type of research. Its advantages and disadvantages have also been pointed out by Dr. Briggs.

One major drawback of the experimental approach to the solution of a problem is that in many of the studies where this type of research is employed the experimental group always or nearly always wins. No one seems interested in doing such experiments over again to see whether the same results can be obtained. And where such studies have been made to check, by doing the experiment over again, I believe the facts will show that for the most part a negative result has been found to obtain.

Again, we often find that certain experimental studies, good as far as they go, but by no means final, need to be carried on to see whether the original findings point to other and more important implications. For example, Seidlin's study "A Critical Study of the Teaching of Elementary Mathematics" should be extended to include a larger number of cases from other parts of the country so as definitely to ascertain whether the teaching of mathematics in the colleges is generally ineffective.

It is certain that many teachers of algebra today are drilling the pupils in some topics far beyond the stage of diminishing returns. Pencil and paper is relatively cheap, but grey matter is expensive. We ought to find out how long it takes to teach certain topics to a certain desirable degree of mastery. How long, for example, should it take to teach a normal group of ninth grade pupils in algebra to factor the difference of two squares? The answer to such questions can be made relatively as precise as the measurement of the length of a room.

5. Philosophical Research.

This type of research is perhaps the best illustrated in my field by Everett's study on "The Fundamental Skills of Algebra." Obviously, Dr. Everett could have collected a great deal of data, and could have worked out some correlations, but he did not choose to do so. Nevertheless his study has probably influenced the teaching of algebra more by leaving the statistics out. At least his dissertation has been most helpful to teachers of algebra.

I look forward to the time when it will be possible for a student of vision and intellect to get certain manifestations of either or both of these qualities into his Ph.D. dissertation. Many of the contributions still to be made in improving instruction in the schools will only be done after careful and prolonged thought by frontier thinkers.

CHOICE OF A TOPIC

What kind of research topic should one choose? First of all it should be a problem in which he is interested and the solution of which is vital. No one should start a problem unless he is interested or is sure he will become interested in it. Ideally the

topic should be chosen by the student and not by his professor. Although some assigned topics of importance have turned out well because the student who took them happened to like them, it is better for a student to choose a topic which involves some topic he has a burning desire to solve. It may be that "The student may supply evidence of as high a degree of originality in the discovery of his problem as in its final solution."[2]

What then are some of the topics that still offer great possibilities to whose who wish to have topics suggested to them? I will give them under the four headings given at the beginning of this discussion.

I. Objectives.

This part of the work is so fully treated in the literature that I will not take time here to go into it. Anyone who considers any list of objectives could easily add to or subtract from such a list according to the needs of his local situation. Moreover, no list will satisfy everyone if all possible needs are considered.

II. Materials of Instruction.

1. Mathematics for gifted children.

The most retarded child in the American public school system is the gifted pupil or the pupil of scholarly mind. What course can we give him that will best meet his needs? Are the present syllabi and textbooks adequate for teaching gifted children? If not, how can we proceed to organize a better course?

2. Mathematics for slow moving groups.

The problem of what mathematics to teach to pupils of below average ability is becoming very acute. Shall we try to find out what mathematics he can learn (regardless of its content value) or shall we first make a plan and then choose the content accordingly? These are questions that must be solved very soon if progress is to be made. The right of a pupil to be happy in his school work demands that we do something to change the intolerable condition that exists in most schools, particularly in the larger cities, among the pupils of lower ability.

3. General mathematics versus traditional mathematics.

After years of careful experimentation and evaluation general mathematics seems to have established itself to be at least as good as if not better than the traditional approach. What further studies will help to direct our steps in this regard?

4. Correlation of mathematics with other great fields of knowledge.

The studies by Congdon, "Training in Mathematics Necessary for College Success," Fagerstrom "Mathematical Facts and Processes Prerequisite to the Calculus," Brown, "Mathematical Difficulties of Students of Educational Statistics," and Helmich on "Mathematics and Social Studies," are good examples of the type of study that is helpful here. We need further studies related to other fields such as art, music, and the like.[3]

III. Methods.

1. Of teaching.

a. Class size.

What is the best sized class for proper instructional purposes? The last word has not been said in attempting to answer this question. There is a fruitful field for research here.

b. Homogeneous versus heterogeneous classification of pupils for teaching purposes.

This problem is closely related to the problem of teaching gifted and slow moving groups. Homogeneous classification is the first step toward individualized instruction. Should we take it?

c. Induction versus deduction.

Where is the place for each in the secondary school? We already have some studies but they are not conclusive.

[2] Almack, John C. *Research and Thesis Writing.* Houghton Mifflin Company, 1930. p. 37. This is a good reference book for students to consult before choosing a topic.

[3] See Reeve, W. D. "Mathematics and the Integrated Program in Secondary Schools." *Teachers College Record,* Vol. 26; pp. 97–507.

d. Teaching training.

There are numerous problems here that cry for solution, but the call has scarcely been heard. Bond's study "The Professional Treatment of the Subject-matter of Arithmetic for Teaching Training Institutions" and Christofferson's study "Geometry Professionalalized for Teachers" are efforts to throw some light on the question of the training of teachers both new and experienced.

2. Of learning.

This field at the present ime offers one of the most promising areas of research. We have a great deal of material in the literature on methods of teaching, but very little has been done to find out howe a pupil learns most easily and most economically. Some of the important problems here are:

a. What difficulties are encountered by pupils in the study of elementary algebra and how can such difficulties be met?
b. A similar question for demonstrative geometry.
c. The problem of transfer.

We know that transfer occurs under certain conditions, but how much transfer is there and under what conditions is the possibility of transfer enhanced?[4]

d. The psychology of repetition and drill in arithmetic.
e. What is the earliest advisable grade in the elementary school in which graphs can be profitably studied?
f. What is the lowest level of ability, if any below which a pupil cannot profit by the study of algebra?
g. A similar question for the study of demonstrative geometry.

In other words who if any should be excused from studying algebra and demonstrative geometry in the secondary schools?

IV. The testing program.

Here is another rich area of investigation. Many teachers who have desirable objectives in mind who have what they consider to be suitable materials of instruction, and who feel competent concerning their methods of teaching fail to a large extent because they do not take the time to test their teaching as they should.

Here are some important questions:

1. What is the importance and place of prognostic tests?
2. What kinds of achievement tests in mathematics should we use and when and how should they be administered?
3. To standardize or not to standardize?[5]
4. Mastery tests.

[4] See Orata, P. T. "Transfer of Training and Educational Pseudo-Science." *The Mathematics Teacher*, Vol. 28, pp. 265–289.

[5] See Reeve, W. D. "Educational Tests—To Standardize or Not to Standardize." *The Mathematics Teacher*, Vol. 21, pp. 369–389.

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Different Kinds of Equality*

By C. C. MacDuffee
Ohio State University, Columbus, Ohio

I. When Thomas Jefferson suggested that all men were created free and equal, he raised two interesting questions. The first question, "What is freedom?" is at present being given some thought by the political scientists, so it seems only fair that we should do our part by attempting to answer the second question, "What is equality?"

The concepts of equality, identity and equivalence have been employed with various shades of meaning since man began to speculate. Philosophers have argued and discussed these concepts at great length. We shall avoid being overwhelmed in this philosophic discussion by refusing to enter into it.

A single instance of these early attempts to define equality will perhaps be of interest, because it was made by a mathematician whose name is familiar to everyone, namely Leibnitz.[1] He stated that "two things are called equal if, in every expression, one may be replaced by the other." Commenting on this definition of Leibnitz, the philosopher Husserl[2] said "it is evident that this is the definition of identity rather than of equality."

I believe that a mathematician of today would find more than this to criticize in Leibnitz' definition. In particular, the word *may* needs elucidation. The evident meaning of his definition is, "Two things are called equal if, upon substituting one for the other in any expression, the new expression is equal to the original expression." Thus equality of *things* is reduced to the question of equality of *expressions!*

It seems clear that Leibnitz' statement is rather a definition of mathematical expression than a definition of equality. A mathematical expression in order to be well-defined must certainly yield equal numbers when equal numbers are substituted into it.

We shall now resist the temptation further to criticize eminent scholars who are not present to defend themselves, and come to the main point of this address. In the *Principia Mathematica* of Whitehead and Russell (1910), the concept of identity is discussed at considerable length. Quite a number of postulates or directives are given. The first of these are substantially as follows:

I. Given any two elements x and y, either $x=y$, or $x\neq y$. (The relation is determinative.)

II. $x=x$. (The relation is reflexive.)

III. If $x=y$, then $y=x$. (The relation is symmetric.)

IV. If $x=y$ and $y=z$, then $x=z$. (The relation is transitive.)

I do not mean to imply that these postulates are original with the authors of the *Principia*. There is evidence of a long period of evolution. Thus twenty years before the *Principia* appeared (1890), Schröder[3] gave II and IV as characteristic properties of the relation $x\leqq y$, and then stated that if III also holds, then $x=y$.

But the significant statement to be found in Whitehead and Russell, the statement which is to be the theme of this address, is a footnote appended to the four properties listed above. It says,

We shall find that no new definition of the sign of equality is required in mathematics: all mathematical equations in which the sign of equality is used in the ordinary way

[1] Leibnitz, *Opp. phil. Erdm.*, p. 94.

[2] E. G. Husserl, *Phil. der Arith.* I (Leipzig 1891), p. 104.

[3] *Vorlesungen der Algebra der Logik*, Teubner 1890, I, p. 184.

* An address presented to the National Council of Teachers of Mathematics at Pittsburgh. December 29, 1934.

express some identity, and thus use the sign of equality in the above sense.

I shall do my best to convince you that the above statement is substantially correct in the sphere of elementary mathematics. But before doing this, I should complete the historical picture by saying that these postulates, or definitions, or characteristic properties, or directives for equality, as you please to call them, are finding favor among prominent mathematicians. They are to be found, for instance, in the algebra of Haupt,[4] and in various papers in journals devoted to mathematics.

Until quite recently, and I must admit quite generally at present, it has been customary among mathematicians to consider the concept of equality as teleological. The difficulty of considering two non-identical expressions to be equal was avoided by calling them equivalent, putting all equivalent expressions into a class, and considering equality of classes. The two points of view lead to the same end. The point of view which each one of you will take will depend upon a personal element. I much prefer the newer approach because it removes the concept of equality in mathematics from the realm of the teleological, as the concepts of *point*, *line*, *number* and the like have been removed during the last century.

II. Let us first see how ordinary plane geometry looks from this point of view. Our elements are geometric figures composed of points and lines. Since we are dealing with intuitional geometry, we shall take for granted the concept of superposition. Two figures will be called equal (or congruent, if you wish) if one may be rotated and translated until it is brought into coincidence with the other. Given any two figures, it is immediately evident that either this can or cannot be done, so Property I holds; that every figure can be superposed upon itself, so II holds; that if one figure can be made to coincide with a second, then the second can be made to coincide with the first, so III holds: finally, that if one figure can be made to coincide with a second, and the second with a third, then the first can be made to coincide with the third, so IV holds. The geometry of congruent figures is the study of those properties which remain invariant under the relation of congruence. Thus, *if the figure composed of two sides and the included angle of one triangle is congruent with the figure composed of two sides and the included angle of another triangle, then the entire triangles are congruent.*

But plane geometry is concerned with other equality relations than simple congruence. The relation of similarity likewise satisfies all four of the properties of an equality, and the geometry of similar figures is a distinct type of geometry. The geometry of figures of equal area is another type also included in "ordinary plane geometry."

In 1871 Professor Felix Klein defined a geometry as the study of the invariants of a group of transformations. Now a group is a set such that

1. The resultant of two transformations is a transformation of the set,
2. The associative law holds,
3. There is an identity transformation,
4. Every transformation has an inverse.

The euclidean transformations form a group, and so do the similarity transformations. Thus the geometry of congruent figures and the geometry of similar figures are instances of geometries as defined by Klein. But there are many other groups of transformations known, and from them we have the less familiar types of geometry—projective, non-euclidean, differential, algebraic, and the like.

The resemblance between the definition of group and the definition of equality is more than superficial. It would take only a few moments to show that equivalence relative to a group of transformations is an

[4] O. Haupt, *Einführung in die Algebra* I, Leipzig 1929.

O. Ore, *Bull. Amer. Math. Soc.* 37 (1931) p. 538.

equals relation. Of course the unrestricted converse, that every equals relation is an equivalence relative to a group is not true, for there are a few branches of mathematics which the geometers have not yet claimed as their own. But it is true that a relationship defined by means of equivalence under a set of transformations is an equals relation if, and only if, the set is a group.

III. Algebra furnishes just as many illuatrations of the appropriateness of this point of view as does geometry. Let us consider, for instance, the introduction of negative numbers. We shall assume that we have the positive numbers and 0, and that we know what we mean by equality, addition and multiplication of these numbers. We shall now consider a new mathematical system composed of the pairs of numbers (a, b) where a and b are positive or zero—positive for short. We define the equality of such pairs in terms of the equality which we know as follows:

$$(a, b) = (c, d)$$

means that $a+d=b+c$. It can be shown in a moment that this definition of equals satisfies the conditions of Part I.

We can see that every pair (a, b) is unequivocally equal either to a pair $(c, 0)$ or to a pair $(0, d)$ according as $a>b$ or $a<b$, or equal to $(0, 0)$ if $a=b$. If $a>b$, then $a=b+c$, which is the condition that $(a, b)=(c, 0)$. If $a<b$, $a+d=b$, which is the condition that $(a, b)=(0, d)$. Finally, $a=b$ is the condition that $(a, b)=(0, 0)$.

Addition and multiplication are defined

$$(a, b)+(c, d)=(a+c, b+d),$$
$$(a, b)\times(c, d)=(ac+bd, ad+bc),$$

entirely in terms of addition and multiplication of positive numbers. It can be readily shown that the sum and product are well-defined—that is, if (a, b) or (c, d) is replaced by another number-pair equal to it, the sum (product) is merely replaced by another number-pair equal to it.

These definitions of addition and multiplication appear arbitrary, it is true. Their justification lies in the fact that when addition and multiplication are so defined, they are associative and commutative, and multiplication is distributive with respect to addition. When these laws hold, all the laws of arithmetic hold, and we have a mathematical system which is really useful.

In particular,

$$(a, 0)+(c, 0)=(a+c, 0),$$
$$(a, 0)\times(c, 0)=(ac, 0),$$

so that if we let the number a correspond to $(a, 0)$, we see that this correspondence is an isomorphism under both addition and multiplication. In other words, we see that the number pairs (a, b) contain a subset of pairs $(a, 0)$ which are abstractly equivalent to the set of positive numbers a. We shall therefore denote the number $(a, 0)$ by the symbol a.

The numbers $(0, b)$ do not correspond to positive numbers, and therefore the set of pairs (a, b) is a proper extension of the set of positive numbers. From the definition of addition,

$$(a, 0)+(0, a)=(a, a)=(0, 0)$$

or

$$a+(0, a)=0.$$

The numbers $(0, a)$ are, then, of a new type which we denote by $-a$. They have the property that $a+(-a)=0$.

Plainly we have not assumed the existence of the negative numbers—we have proved their existence by exhibiting a representation of them in terms of pairs of positive numbers.

This treatment leads to a wholly satisfactory answer to that troublesome question of why $(-1)(-1)=1$. From the definition of multiplication

$$(0, 1)\times(0, 1)=(1, 0).$$

I shall close with one more illustration—the extension of the real number system to the complex number system. Consider all polynomials

$$a(x)=a_0+a_1x+a_2x^2+\cdots+a_nx^n$$

of all degrees, the coefficients being real. Two polynomials $a(x)$ and $b(x)$ are defined to be equal if $a(x)-b(x)$ is exactly divisible by x^2+1. It is readily shown that the

postulates for an equals relation are satisfied. Evidently every polynomial not 0 is equal to one and only one polynomial of degree ≤ 1.

Addition and multiplication are defined in the usual way for polynomials. Thus,

$$\begin{aligned}&(a_0+a_1x)(b_0+b_1x)\\&=a_0b_0+(a_1b_0+a_0b_1)x+a_1b_1x^2\\&=a_1b_1(x^2+1)+(a_1b_0+a_0b_1)x+a_0b_0-a_1b_1\\&=(a_1b_0+a_0b_1)x+a_0b_0-a_1b_1.\end{aligned}$$

The polynomials of degree zero form a subset of our new system which is isomorphic with the real numbers with which we started. The polynomials of degree 1 are numbers of a new type. If we set $a_0=b_0=0$, $a_1=b_1=1$ in the last equation we have

$$x^2=-1.$$

If we use the letter i in place of the letter x, the number system appears in a form more familiar to us.

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A Redefinition of Secondary School Mathematics

By LEROY H. SCHNELL
Roosevelt School, Michigan State Normal College
Ypsilanti, Michigan

MATHEMATICS is alarmingly on the defensive. Much that has been traditionally sacred to us is rapidly disappearing from the curriculum. Our past security is vanishing into thin air. The writers of the National Committee Report admitted that, "the subject must, like others, stand or fall on its intrinsic merit or on the estimate of such merit by the authorities responsible at a given time and place."[1] Seven years later Betz stated that, "Geometry shares with Algebra and Latin the fate of being on the defensive."[2]

The forces which have precipitated the crisis in our field are several. The question concerning who should take mathematics, and in what amount, and of what kind, results from several causes. The current depression has entailed retrenchments to such an extent that every subject that could not be adequately defended, or loudly enough "ballyhooed," was eliminated from the curriculum or painfully restricted. Out of the depression came a social consciousness which projected one segment of our curriculum, namely, the social studies, into a position of prominence with a consequent loss of prestige to the conventional subjects. Then the colleges and universities, which had long dominated secondary schools with stultifying entrance requirements, awakened from their lethargy and swung into line with progressive educational principles. Some have accepted, and others are considering the acceptance of, candidates for admission who have sampled generously the broad curriculum of the modern high school rather than having directed their energies to the few conventional subjects. These changes, which may lead to our ultimate salvation by arousing *us* to the realization of our insecurity, are, however, to our immediate disadvantage, for high school administrators, having been given more leeway, will not be slow to liberalize still further the secondary school program.

An attack on the general educator, the public school administrator, or the accrediting agencies for a gross ignorance and inability to see the enduring beauties of mathematics will not strengthen our position and may serve only to call attention to the fact that our house is not in order. Furthermore it will alienate the sympathy and cooperation of those whose support we need most. It is our business to convince them that it is imperative as never before, in this era of social transition and insecurity, that our schools shall produce boys and girls capable of doing the kind of critical and straight thinking that we have always complacently contended mathematics will help them to do. This, perhaps, is an overstatement of the true issue, for schoolmen in general are aware of this necessity, but it remains for us to convince them that mathematics has a vital part in realizing such an objective. We may have performed our task poorly in the past but what will be the result as more and more we find mathematics culled from the curriculum to be supplanted by subjects in the formative stage of development? Of one thing we can be sure, and we must convince those who censure us of this fact: *only as it is capable of doing the kind of thinking and acting, that we are entitled to call mathematical, will our society achieve intelligent progress.* The issue cannot be evaded longer. If mathematics is to retain a respected place in the secondary school,

[1] *Report of the National Committee on Mathematical Requirements*, p. 32.

[2] Betz, William, "The Transfer of Training, with Particular Emphasis to Geometry." *Fifth Yearbook of the National Council of Teachers of Mathematics*, 1930, p. 149.

we must establish it in that place ourselves. If we succeed it will mean that we have demonstrated an ability to meet the needs of a changing social and educational order.

We have equipped our critics with potent weapons. A review of mathematics textbooks would lead one to believe that we still maintain an untenable position in favor of faculty psychology. True, some authors attempt to provide opportunities for generalizations which *may* lead to some transfer, but where these attempts are made they are often rendered impotent due to the fact that they rarely transcend subject matter carry-over or because the new situation to which a principle is applied is entirely out of focus with child interest or necessity. If the foregoing is somewhat unfair, we must admit, however, that we have lagged so far behind advances made in scientific educational knowledge that we appear to be incapable of much change or adjustment.

The problem of transfer of training is ever with us and must be considered in any redefinition of our subject. It is true that extensive studies must be made to determine what elements and generalized experiences in mathematics do carry over to non-mathematical situations, but it is likewise true that we have failed to heed the warnings of writers in our own professional literature and that we have likewise failed to apply adequately what we already know or surmise. If the reader will examine the present day secondary school mathematics curriculum in the light of the following quotations, the foregoing points will be established.

In the National Committee Report the statement is made that, "The amount of transfer in any case where transfer is admitted at all, is very largely dependent upon methods of teaching."[3]

In Betz's opinion, "There are excellent reasons why school subjects '*as now organized*' should lead to such a 'small degree' of transfer. We do not secure transfer *unless we train for transfer.*"[4] Also, " '*Scientific methods' thus far, have not convincingly settled the question of the extent of transfer.* Nor have they thrown very much light, if any, *on the vitally important problem of how we can train effectively for transfer.*"[5]

The most recent statement on this topic in our literature is made by Wheeler. "No transfer will occur unless the material is learned in connection with the field to which transfer is desired. Isolated ideas and subjects do not integrate. Learning is not bond-forming. It is an orderly and organized process of differentiating general grasps of situations with respect to experience. The details emerge organized, as they differentiate from previous knowledge, in the face of new situations (not repeated ones)."[6]

Our position will be more tenable if we claim transfer only for cases and situations where elements are actually shown, scientifically, to transfer. Perhaps we are sure in our own minds that the techniques of geometry carry over to social thinking, but our belief is only a rationalization until we investigate and determine which techniques transfer and to what extent and under what conditions they do. We cannot permit any suggestion of a contention for faculty psychology in our defense or an arm-chair contention for transfer of training. Our best defense lies in a redefinition and rebuilding of our subject to meet educational and social needs.

It is astounding that we should hold to the narrow view of our subject that our teaching implies we do. Mathematics is still mathematics when it is stripped of number relations and space concepts. It is more than a bundle of logical facts, skills and devices—it is an intellectual attitude. Stated more specifically, mathematics is a *mode of thinking.* Herein we

[3] *Op. cit.*, p. 95.

[4] Betz, *op. cit.*, p. 167.

[5] Betz, *op. cit.*, p. 170.

[6] Wheeler, R. H., "The New Psychology of Learning." *Tenth Yearbook of the National Council of Teachers of Mathematics*, 1935, p. 239.

may discover a definition of our subject that will vitalize it as an indispensable unit of the secondary school curriculum.

The crux of our present problem lies in the fact that we have been concerning ourselves with the teaching of mathematics *per se* and have given little or no specific attention to the inculcation of mathematical *modes of thinking*. We have crammed children's minds to capacity with the mechanical skills, processes and relationships of mathematics, assuming that in this procedure they would incidentally, or by some psychological hocus-pocus, acquire an insight into the *modes of mathematical thinking* and likewise the ability to apply these modes of thinking to non-mathematical situations. Experience has taught us in no uncertain terms that this is a fallacious procedure.

We must not forget that much of the arithmetic we teach does not function in life; that elementary algebra is largely a fabrication of one academic experience to prepare the child for another equally academic experience. Some of our algebra can easily be defended as useful and educative, but a large part of the present course is time spent in mental gymnastics and busy work. As for geometry, consider the following quotations.

Eric T. Bell, in a recent volume[7] makes two pointed criticisms which should jar the complacency of every teacher of plane geometry. He says, "A diluted sort of Euclid, looser than his loosest, is one of the main-stays of American Education today. It is supposed to quicken the reason, and there is no doubt that it does *in the hands of a thoroughly competent and modernized teacher*,[8] who lets the children use their heads and see for themselves exactly how nonsensical some of the stuff presented as 'proof' really is. But, as nothing else even half so good is offered in the way of deductive reasoning in school, we must not be too hard on what is actually handed out. The great miracle is that there are not a hundred million gullible boobies in America, eager to swallow all the latest and craziest speculations, instead of the negligible few there are."

He adds the parting shot that "if school children fail to get some conception of geometry and close reasoning out of their course in 'geometry' *they get nothing, except possibly a permanent inability to think straight and a propensity to jump to conclusions which nothing in reason or sanity warrants*."[8]

As to mathematics in general, Professors Wheeler and Perkins state, "Everywhere, mathematics is the final source of appeal for accurate work. It is deplorable that, as a subject, it is unpopular and considered to be difficult; more deplorable still that school systems do not require more of it and make it sufficiently interesting to appeal to the average student. It can be done, and the right kind of teacher has always been doing it."[9]

Education in a democracy must develop proficiency in the three following activities: in seeing and formulating worthwhile problems; in assembling and organizing data having a direct bearing on these problems; and in acquiring techniques that will insure adequate solutions for them. In the average mathematics classroom children are given experience in these activities in the most elementary space or number situations and since mathematical *facts* are simple in nature the child is not motivated to extend his intellectual powers beyond the mastery of a series of frequently unmeaningful facts and processes. He quite naturally becomes a manipulator and does not recognize that the modes of thinking, by which these facts are revealed, are of vital importance to his intellectual growth. We, ourselves, have failed to recognize, or at least to

[7] Bell, Eric T., *The Search for Truth*. New York: Reynal and Hitchcock, 1934, pp. 117–119.

[8] Italics mine.

[9] Wheeler, R. H. and Perkins, F. T. *Principles of Mental Development*. New York: Thomas Y. Crowell Company, 1932. P. 480.

apply, the fact that **wherever, and in whatever context, the mathematical modes of thinking are incident to any of the three foregoing activities, mathematics is being employed,** even though the situation may transcend the use of number relations, space concepts, and Q.E.D.'s.

Professor Young says,[10] "But the facts of mathematics, important as they are, are not the strongest justification for the study of the subject by all pupils. Still more important than the subject matter of mathematics is the fact that it exemplifies most typically, clearly and simply certain modes of thought which are of utmost importance to everyone.

"One of these modes of thought is the ability to grasp a situation, to seize the facts, and to perceive correctly the state of affairs. This is prerequisite to success in any occupation, but it is a hard thing to do in actual life. . . . It is a long way from the simple system of elementary truths of mathematics, whose present form is the result of centuries of polishing and remodelling, to the complex facts of the social organism."

The mathematics which we offer for common consumption in the secondary school must emphasize *modes of thinking*, WHERE THE RESULT OF THINKING TO BE DONE IS IMPORTANT. The content must be from all subject matter fields. We must extend the application of our *techniques* beyond the simple cases of mathematical fact. The techniques which we must train the child to apply should include experience in discovering underlying principles and obscure facts, in sensing implied assumptions, in applying logic, in reasoning from data with any set of assumptions, in seeing the dependence of conclusions upon the assumptions, in judging the validity of conclusions, in analyzing, in generalizing, in expressing oneself quantitatively and symbolically, in interpreting symbolism and abstractions, in organizing and arranging ideas concisely, in expressing oneself with precision and accuracy, in sensing and interpreting functional relationships and in developing a wholesome challenging attitude and the attitude of inquiry. Proficiency in these activities is of more importance than mastery of any of the mechanical procedures of the subject, and whenever a student employs any of them he is using mathematics whether it is a traditionally conceived mathematical situation or a distinctly non-mathematical one.

Raye Roberts Platt, Secretary of the American Geographical Society, while a student at Ypsilanti, was asked why he was wasting his time on mathematics when he demonstrated such talent for writing. His reply was, "You say that you like the way I organize my materials. Well, I learned that in geometry." Some mathematics teacher had *taught for transfer* or else this is another case where a student "caught on" in spite of the teaching. Regardless, this student had learned to organize and arrange his ideas concisely and also doubtless saw the counterpart of mathematical logic when he was striving for *coherence* and *unity* in written expression.

As mathematics teachers we may not be interested, primarily, in the answers which our social "scientist" colleague gets, but we must be vitally interested in *how he gets there* and in subjecting his conclusions to tests of validity. When we capitalize these interests functionally we are employing mathematics.

The fact that the social studies contingent so rarely apply mathematical modes of thinking in the solution of their problems accounts, to a large degree, for the inadequacy of the solutions proposed and the fact that some of these people are ready to resort to indoctrination. But we are equally to blame. They have tried to bring into the classroom vital social problems for students to grapple with while

[10] Young, J. W. A. *The Teaching of Mathematics*. New York: Longmans, Green and Company, 1906. Pp. 17–18.

we have pedantically denied the same students the proper techniques they needed for attacking these problems intelligently. It is time to get together on common ground. They know, and many of them will admit it, that in their subject there are practically no fundamental *assumptions* upon which they can get reasonable agreement, and furthermore that much of the reasoning which they employ in the "solution" of their problems is so wishful and disjointed that it cannot long hold their subject together. They need to *objectify* their materials, envision *precise ends* to be striven for and acquire mastery of the *techniques of critical thinking* which are necessary for the realization of their established objectives. As mathematics teachers we have a vital part to play in this program.

There are endless opportunities to integrate mathematics and the other subjects of the curriculum to the end that both will become more meaningful and educative. Every subject in the secondary school involves either the content of mathematics or the mathematical modes of thinking. It remains for us to utilize this fact in a practical, functional and non-academic manner. We must redefine the purposes of mathematics and assemble the materials that will yield the desired results even though most of our traditional mathematics falls by the wayside. To realize our aims we must stop the pretense of giving students the final answers and instead give them a sound device for *finding sane answers.*

The mathematics curriculum, as now constructed, cannot effect any broader aim than subject matter acquisition; it has been developed primarily from that point of view and teachers are trained and equipped with the artifices for achieving it. We have made our subject more human in the last generation by emphasizing the importance of methods and by applying better psychology in our teaching. Adept and competently trained teachers have always attempted, and to a considerable extent have succeeded in achieving, the aims implied in this paper. But there is too vast an army of professionally indigent teachers of mathematics to stop here. Even though they may be inadequately trained and have no interest in or feeling for mathematics, they are with us always and are *our* problem.

To redefine and rebuild mathematics to meet the needs that have always existed, but which have been forced to our attention with greater emphasis recently, will necessitate a reorientation for us all. We have been guilty of evasions in the past for which no defense is to be found. In avoiding the fundamental issues of mathematical thinking, textbook writers have perhaps proceeded on the theory that high school students are too immature to go to the roots and comprehend the meanings. For example: there is no geometry text on the market today that goes the whole way in making honest and clean-cut statements as to what the student can expect the subject to do for him or about the assumptions upon which all the reasoning is to be based. In most cases students are permitted, or even encouraged, to believe that the assumptions are God Given and Universal Truths. Why not tell children candidly that these assumptions are only man's feeble attempts to express what *seems* obvious in terms of his experience? Why go any farther than to tell them that here are the rules of the game we are going to play and that most of the assumptions *seem* to make good sense? Why not dispel the idea of finality and omnipotence about mathematics by injecting the proper degree of uncertainty at the beginning? We will admit that mathematics is fundamentally experiential and assumptive. There is no reason, then, why we should not acquaint our students with the true nature of the subject, for when we do not approach our teaching candidly the child later finds himself in a contradiction of ideas, and mathematics pays the penalty.

The present and oncoming adventurous

generation refuse to accept established institutions and custom on the sole grounds that they are traditional, "proved" and safe. They are a challenging, and in a sense, a thinking crowd. The platitudes which served in the past will not suffice now. These youngsters will not cherish, honor and respect anything with the slightest trace of evasion. Absolute honesty is the keynote to survival.

Possibly it is true that not all children can profit from a curriculum in which techniques of thinking are to be emphasized but we will never know how much *real* thinking even slow children can do until we try them out. It is folly, of course, to expect subnormals to accomplish tasks beyond their capacities as we have done in the past. Most failures can be traced to this error, which unquestionably has helped precipitate the present crisis. Veritable armies have learned to dislike mathematics because they could not succeed in it. It is our job to convince moron and genius alike that mathematics is an indispensable tool that will help them do the things they *can*, and *want* to do, *well*. We must stimulate our students to approach their study of mathematics with keen curiosity and a challenging attitude, offering them experiences that will enable them, one and all, to emerge with a feeling of respect for MAN'S most PRECISE TOOL OF INQUIRY.

We face the very real necessity of redefining secondary school mathematics in keeping with its real value; of insisting on retention of the traditional mathematics for which a substantial defense can be found; of securing reasonable agreement to a new definition and its implications; of developing the new materials with which to work; and of re-educating teachers to adjust to a new order. These changes cannot be accomplished in a year nor in ten, but they will never result until we start some *concerted* action not unlike the National Committee study of a decade ago.

We cannot expect that average classroom teachers will see where their subject "ties into" others, or that they will take the time, or that they will be equipped, to arrange such a program. Unfortunately the norm of our teaching practices is low but the rank and file of the teaching profession is capable of making changes provided the leadership is afforded.

This paper can be summarized into one challenge: mathematics must give the student a tool to facilitate clear, accurate, logical, and quantitative *thinking* IN EVERY PROBLEM SITUATION and, at the same time, must whet his curiosity so that he will see more of the problems which abound. If it fails herein, then it has no claim to the prominent place that it has held in the curriculum.

THE EDUCATION of the whole people in a republican government can never be attained without the consent of the whole people. Compulsion, even though it were a desirable, is not an available instrument. Enlightenment, not coercion, is our resource. The nature of education must be explained. We cannot drive our people up a dark avenue, even though it be the right one; but we must hang out the starry lights of knowledge about it and show them not only the directness of its course to the goal of prosperity and honor, but the beauty of the way that leads to it.—HORACE MANN.

Recreational Aspects of Mathematics in the Junior High School*

By Anna R. Meeks
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It was with some surprise and considerable consternation that I received my assignment of the topic, *Recreational Aspects of Mathematics in the Junior High School*, for I have been, as I suspect most teachers have been, a slave to courses of study, and the recreational side of mathematics has been as sorely neglected as the recreational side of the average teacher's life. Several readings of the topic convinced me that I was not even sure of its meaning; so I made my own interpretation and proceeded to try to discover the answers to my own questionings. That study proved enlightening and interesting; it is my purpose to pass on to you some of the practical suggestions which I found in various sources.

The study began with interviews of friends who are fairly well removed from school-room contacts with mathematics. Usually I began by telling them of the meeting, and of the speech which I was obliged to make; all of which was most interesting until I propounded my question, "What are the recreational aspects of mathematics?" This invariably brought forth the usual replies such as "There is no such thing," or "Do you mean to tell me you are going to attempt a speech on such a topic?" Certainly we have failed to give a pupil a fair mathematical experience if he leaves the subject feeling that way about it. Perhaps some of the stigma that attaches to our subject would not be there if we had given due consideration to the matter. That a stigma does attach is shown by the public's evident lack of interest in the mathematics section of a school exhibit. Year after year we carefully prepare an exhibit of the work of the mathematics department only to have visitors literally "hold their noses" as they speed by lest they be contaminated by too close contact with the despised subject. I do not believe that our exhibit is inferior to that of other departments, but rather that our failure in the past to relate mathematics to the recreational interests of the pupil has made mathematics a remote uninteresting subject to be left for the musty old scholar.

What recreational values are to be found in mathematics? In an interesting article on "Why Study Mathematics?" F. L. Wren[1] lists among a great many other worthy reasons a number of recreational values: (1) "The proper appreciation of beauty of form is a contribution that mathematics makes toward the proper use of leisure time; (2) The expansion of man's field of thought to include a familiarity with the technique of mathematics provides him with a means of keeping in touch with a certain amount of scientific progress; (3) For the mathematician who is interested in the study for its own sake there is the joy and beauty of mathematical creation just as for the musician there is the joy of musical composition; (4) When looked at purely from the standpoint of play, one must realize that number is fundamental to almost all games in varying degrees of complexity, and there are many amusing mathematical recreations that can afford moments of interesting and entertaining diversion."

My problem, as I see it, is to find those recreational activities peculiar to the junior high school group which will promote a healthy interest in mathematics, develop a worthy use of leisure time, and

* Paper read at the Spring Meeting of the Mathematics Section of the Maryland Teachers Association at Hagerstown in April, 1935.

[1] Wren, F. L. "Why Study Mathematics?" *The Mathematics Teacher*, Vol. XXIV, December 1931, pp. 473–482.

contribute generally to the aims of secondary education. These activities divide themselves into two groups, club and extracurricular activities, and class room activities.

The use of mathematical puzzles and games in the class period will serve to pick up flagging interests and to motivate drill work, and to provide enriched activity for the pupil who can do his work in less time than the average member of the class. The library shelves or tables should contain some of the following: *Mathematical Recreations* by Dudeny, Lick, Cavendish or Schubert; *Number Stories of Long Ago, Number Games, and Number Rhymes* by Smith; *A Boy's Own Arithmetic* by Weeks. It is not sufficient to place the books on the shelves; the teacher must encourage the pupils to use them. A file of puzzles from the newspapers and magazines should be kept. It is interesting to note that many of these bear a strong resemblance to the types of questions suggested in the *Tests of Mathematical Relations* given in the Ninth Yearbook of the National Council of Teacher of Mathematics on *Functional Thinking.* If you are interested in finding a game which has all the elements of play but gets in a terrific amount of drill, you will find it in the algebra baseball game described in *The Mathematics Teacher* for May 1930.[2] This game, which uses baseball terminology and translates rules of baseball into terms of drill procedure, will capture the attention of those boys and girls who hate drill of any kind, and it will make better students of all the members of the class. Best of all, this game suggests other types of games which the creative genius of any teacher can produce to fit those types of drill needed in her class.

Why have a mathematics club? Would it be popular? When should it meet? These and countless other questions suggested themselves to me, and if I hoped to find through a study of club reports that mathematics clubs are unnecessary appendages I was surely doomed to be disappointed. Norman Anning[3] of the University of Michigan has laid down the theorem, "If a mathematics club is alive and alert, then it will be of value to all members of the class." It provides an enriched mathematical experience for pupils which a course in mathematics can hardly provide. Reports of clubs in *The Mathematics Teacher* certainly indicate that they are popular, for most schools have found it necessary to find some fair means of limiting membership. When should the club meet? If the school has an activity period, the problem is settled; if not, it should meet in a class period. Certainly a club should not be an after school affair, for pupils who ride on school busses would be excluded from membership, and the tired teacher already has too many things demanding her attention after the hour of dismissal.

We must turn to Norman Anning[4] again for advice concerning matters of organization. The faculty adviser, he tells us, "should be like the spare tire, inconspicuous but ready when needed." The choice of officers, the name and the constitution should come from the pupils. The president should preside, the secretary record and the committees commit." The club should have its own student critics. Teacher guidance and not teacher domination should be the rule.

What can we do? A great many types of interests can be developed, for the club activities should include: dramatics, field trips, projects, reports, reading together, the making and the singing of songs, and the playing of games. Pupils adore to be in plays, and here is a chance for informal dramatic work. To begin with plays to be found in *The Mathematics Teacher* and other sources could be used, but soon the

[2] "Algebra Baseball Game," *The Mathematics Teacher*, Vol. XXIII, May 1930, pp. 317–320.

[3] Anning, Norman, "High School Mathematics Clubs," *The Mathematics Teacher*, Vol. XXVI, February 1933, pp. 70–75.

[4] *Ibid.*

pupils will want to write their own plays. Dramatic work need not be confined to club programs; it can be introduced into regular class room work. "The Eternal Triangle," by Raferty,[5] using three boys, is an excellent introduction to a unit on indirect measurement. Pupils like to ask riddles and to play number games. The club might sponsor tournaments in chess and dominoes. An alert club would would enjoy making a mathematics room have a mathematical atmosphere. Here the artists could share in the making of posters and charts.

It is with reluctance that I mention the possibility of a newspaper, but a very little energy expended in this direction could do much to make a school mathematically minded. Brief reports of the activities of the club, puzzles, games and riddles, and short essays on famous mathematicians or phases of the history of mathematics would make it all seem real; and the interest thus aroused would express itself in an increased enrollment in elective mathematics courses.

The club should provide an opportunity for the pupil to become acquainted with interesting personalities. How much different would be the pupils' attitude toward mathematicians if the members of the club read together *Alice in Wonderland* thinking of its author as a mathematician of note! His "ambition," "distraction," "uglification," and "derision"; and his "take a bone from a dog and what remains?" have endeared him to many boys and girls, but I wonder how many of them think of him as the famous mathematician, Charles L. Dodgson. Here is a splendid chance to enjoy his letters to the original Alice; to chuckle over his refusal to accept the millions of hugs and kisses because it would consume more time than he had to live. But if you would discover all the possibilities in this field you must read the most interesting article on the life and work of Charles Dodgson in *The Mathematics Teacher* for January 1932.

Has mathematics a recreational aspect? I am sure you will agree with me that the possibilities are almost unlimited, and that any teacher using the skill and ingenuity peculiar to teachers can use this side of mathematics to make a real contribution to the enrichment of the lives of the boys and girls in our schools. Our favorite subject will have gained in prestige, but better still the pupils will have richer and fuller personalities because they have come to know and respect that greatest of all the arts—mathematics.

[5] Gerald Raferty, "The Eternal Triangle," *The Mathematics Teacher*, Vol. XXVI, February 1933, pp. 85–92.

ALL THINGS ARE NUMBERS

NEARLY 2500 years ago the Greek philosopher and mathematician Pythagoras conjectured 'that all things are numbers.' His bold affirmation was based upon a slight knowledge of the heavenly bodies, of the regular geometric solids, and of the stretched musical string. Yet all physical science since his day has attested the truth of his magnificent generalization by establishing the universality of numerical law. By adopting the Pythagorean outlook and applying it to our own inconceivably more body of knowledge we too may hope to obtain new enlightenment as to what science suggests.—BIRKHOFF, GEORGE D., "Science and Spiritual Perspective." Century Magazine, June, 1929.

Activity in Mathematics—The Slow-Moving Pupil

By VIRGIL S. MALLORY
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A GREAT deal has been written in current educational publications about the slow-moving pupil. It is generally recognized that his slowness in academic subjects is not in general compensated for by special ability in the arts and crafts. In mathematics the slow pupil becomes a serious problem in the ninth grade when he attempts to take the ordinary course in college preparatory algebra. Probably the following general statement of the problem will be accepted by most mathematics teachers.

1. Rapid growth of the high school population has resulted in a large number of ninth grade pupils who have no interest in, no capacity for, and no desire to do worth while work in conventional high school mathematics. Not only have they not an adequate mastery of the fundamental skills and processes in elementary arithmetic; they do not have the mental equipment essential for success in algebra or geometry.

2. Attempts to modify and dilute present courses in algebra and geometry to fit their tastes and abilities have the tendency to remove all incentive and challenge from the pupil of average or superior ability who is not only capable of successfully pursuing such courses but who is also capable of making excursions into the field of higher mathematics and its related fields.

3. As long as these slow-moving pupils are in high school, poorly equipped for the quantitative aspects of their future lives, the high school must provide materials of instruction adequate to fit their needs, abilities, and interests. The topics selected must be justified on the basis of recognized criteria and must be taught in a manner that will best accomplish the aims and objectives agreed upon as desirable.

In previous articles,[1] as a result of experiments begun in 1925, the author has arrived at certain general conclusions. These may be summarized briefly as follows:

1. Suitable courses in mathematics should be provided for all pupils in high school, including those pupils who lack mathematical ability.

2. Demonstrated lack of mathematical ability should not be used to excuse pupils from properly selected mathematical courses.

3. The mathematics taught to slow-moving groups should not be confined to "tool" mathematics but should include training in the study of relationships.

4. The method of teaching slow-moving groups must be adjusted to take into account the importance of pupil interest in relation to learning.

5. Pupil activity and laboratory work in mathematics have more importance with slow-moving groups than with other classes.

6. The teacher's personality, his interest in the pupil, and the richness of his background determine to a large extent his success with these groups.

7. The material taught should be so-called general mathematics including social uses of arithmetic, practice in computation, simple algebra, observational geometry, and numerical trigonometry.

8. This mathematical training should extend at least over two years, the ninth and tenth years in the high school.

9. Individual success or failure with such groups should be determined by intellectual growth, not by predetermined standards.

[1] "A Course in Mathematics for Pupils Not Going to College," October, 1932; "Mathematics for the Slow-Moving Pupil," November, 1933.

The author has received requests from many parts of the United States for further details about the materials of instruction and about the most effective methods to be used in teaching these pupils. The following topics are suggested for such a course, with the full realization that others might make a different selection. In view of the necessity for slow, thorough teaching and of the advisability of pupil activity throughout in making situations concrete, there is more than enough material for a year. Selections may however, be made from these topics. While the subject matter is listed under topical headings, the material probably should not be so taught. Thus, while the pupil has not mastered the fundamental skills of arithmetic, it would be difficult to maintain the course suggested at the high level of interest necessary for success, if a great deal of time were spent on indiscriminate drill in computation. It is suggested rather that this work be distributed throughout the course to meet each individual pupil's needs, these needs to be determined by diagnostic tests.

The criteria used in selecting material for the course are:

1. *Every topic selected must definitely and positively contribute to the social, home, or community needs of the pupil.*

2. *Every topic must be of such a nature and presented in such a way as to interest the pupil and to convince him of its value to him; that is, it must be intrinsically interesting and of evident worth.*

3. *The material must be concrete to him and within his ability to succeed with it.*

4. *The material must be of such a type and presented in such a way as to develop desirable habits, attitudes, and appreciations.*

Materials which lend themselves to the attainment of these ends may be listed as follows:

1. *Measurement.* The meaning and need of direct measurement; the measurement of lengths, angles, and areas; how inaccessible lengths may be found by scale drawing; use of the tangent and of similar triangles.

2. *Algebra.* The use of formulas to represent rules and number relations; equations used to simplify the solution of problems; numbers used to represent direction as well as distance—signed numbers.

3. *Graphs.* The use of pictures to represent numbers and number relations; the statistical graph; graphs of formulas; how problems may be solved by graphical methods.

4. *Geometry.* The discovery of number relations, rules, and formulas through geometric drawing; the volumes of solids determined experimentally by making models.

5. *Social uses of mathematics.* This will involve personal, home, school, and community problems. A wide variety of units of work and of problems may be considered here, their selection to be determined by the four criteria given. Some are: making personal accounts and budgets; economy in buying; the cost of waste; buying gas and electricity; buying on the installment plan; problems from general science, from geography, and from the social sciences; how the commercial bank renders personal and community service; the need of savings and how to invest them—the savings bank; protection against loss—insurance.

6. *Arithmetic computation.* To be introduced, according to individual needs determined by diagnostic tests, throughout the course.

The methods used in teaching this material should be those which will adequately attain the desired objectives. Among these objectives may be listed the following obvious *needs* of these pupils which must constantly be borne in mind in each day's teaching:

1. Need for an adequate vocabulary
2. Need for better reading ability
3. Need of ability to follow directions precisely

4. Need for *concrete vital* situations, as opposed to *book problem* situations

5. Need for pupil activity to make situations concrete and vital

6. Need for diagnosis and remedial work in arithmetic

7. Need for immediate and frequent testing to follow the teaching of each small unit of work

8. Need of an appreciation of the pleasure to be found in doing neat, accurate work and the establishment of such habits

9. Need of an appreciation of doing work in mathematics so carefully and of checking so adequately that results are known to be correct. The establishment of the habit of checking

10. Need of an appreciation of the part mathematics has had in the development of civilization

With these needs in mind it is possible to so teach a course in mathematics that the desired ends can be accomplished. For this purpose a modified supervised study or laboratory plan is desirable. It should be said in passing that this supervised study does not imply "busy work" for the class while the teacher does clerical work. It means constant supervision and instruction to make certain that each pupil adequately understands what he is to do and that he takes the pains to do it well. Results, drawing conclusions, and discoveries, should be carefully entered in a note book.

Desirable equipment for the conduct of such an activity class in mathematics, given in the order of its importance is:

Protractor

Rule, divided into 8ths and 16ths

Centimeter rule

Graph paper ruled 10×10 squares to a square inch

Centimeter graph paper

Compasses and dividers

Scissors and heavy drawing paper for making models

T-square, drawing board (12 in. by 17 in. is large enough) and triangles for each pupil; (this set can be obtained for about 90 cents)

Field transit and 50-foot tape for each four pupils.

The success of the use of pupil activity must be judged in the light of the positive contributions it makes to

1. Pupil understanding
2. Pupil interest
3. The attainment of desirable attitudes, habits, and appreciations
4. Pupil growth in the understanding of mathematics and of mathematical methods.

Kinds of activity in presenting the material outlined above together with the special objectives to be sought can be listed as follows:

1. *Making direct measurements.* In this the pupil through activity sees the need for units of measure, studies degree of precision, effects of error, and arrives at mathematical conclusions through experimentation and measurement. He also sees the growth of natural units of measure and civilization's need for standardized units.

2. *Making and reading of functional and statistical graphs.* Picture, broken line, bar, component parts bar and circle graphs, should be related to pupils' interests. Their study should involve considerations of mathematical principles evident in the construction and reading of the graph. Line graphs should be used to show the interpolating of values on both axes, rate of change (slope) and kinds of variation.

3. *Use of graphs in solving problems.* The use of line graphs and of component parts graphs in solving a wide variety of problems should give the pupil the habit of making concrete those mathematical situations which come to him for solution. In doing this it is advisable to go much farther than the ordinary text in solving rate, time-table, and work problems, and in solving problems involving the fractional parts of a whole. Thus fractional relations are made vivid and the pupil's difficulties in doing abstract reasoning are recognized and met.

4. *Making models for three-dimensional figures.* Difficulties which these pupils meet in visualizing relations are removed and mathematical principles are discovered, discussed, summarized, and generalized through experimentation.

5. *Making geometric drawings.* Besides the opportunity for emphasizing habits of neatness, order, and accuracy, the pupil here masters geometric concepts and relations through activity instead of through passive acceptance.

6. *Use of drawing board, T-square and triangles.* These instruments facilitate the pupil's efforts to do neat, careful work. Thus the opportunity is afforded to build up a desire to have all work meet a high requirement of neatness and accuracy. The engineers' training in form in written work as an incentive and aid to accuracy can well be applied to work with slow pupils.

7. *Reading plans and maps and making plans.* Entirely aside from the value to the pupil of the ability to read maps and plans, the greatest value in this work lies in the opportunity of the teacher to develop those desirable traits and habits previously listed.

8. *Indirect measurement.* In connection with the making of plans and with direct measurement out-of-doors, this unit of work never fails to interest pupils. The height of trees and buildings, the width of rivers, the height of mountains, can all be related to the interesting problems of the astronomers. Not only should right triangles be included, but also the finding of areas, angles, and distances in oblique triangles through scale drawings. The principal value of this work lies in the *reality* of the problems. That is, the heights, distances, etc., should be obtained from real outdoor measurements.

9. *Number scale.* In teaching signed numbers the signed number scale should be used for addition and subtraction and the truth of the laws for multiplication and division should be rationalized through real problems.

10. *Making and using formulas.* Many formulas can be made by actual class room experimentation in connection with a table of values. Further activity can be provided through the construction of tables and of graphs.

11. *Activity in personal, home, and community problems including banking, insurance, and savings* can be provided through

a. The making and ruling of ledger accounts, budgets, checks, check stubs, etc.

b. The use of graphical methods in making vivid important relations.

Thus, we have: (1) the setting up of definite, concrete objectives; (2) the selection of materials of instruction to attain those objectives; and (3) the adoption of class room methods and of activities which will make the work concrete and which will aid in attaining the desired objectives. Throughout it all we have a keen appreciation of two facts: (1) the best results can be obtained only if the pupil is inspired and interested; and (2) it is difficult to interest any one in work unless there is a good chance of his being successful in it.

THE INSTRUCTION AND THE DISCIPLINE OF STUDYING MATHEMATICS

So IMPORTANT is mathematics in contemporary civilization that for appreciative values, if for nothing else, it would be an injustice to the learner to let him miss the instruction and the discipline of mathematical study unless he proves himself to be utterly hopeless on these higher emergent levels of learning. . . . It is a notable fact that some of the recent advances in natural science have been made by mathematicians who have worked vigorously and persistently on "If—then" hypotheses as applied especially to astronomy and physics.—BAGLEY, W. C. *Education and Emergent Man.*

Travel Motivates Mathematics

A Unit Based Upon Foreign Exchange

By Eoline Wallace Moore
Birmingham-Southern College

A junior high school teacher who brought back some coins when she went abroad last year helped her pupils to build an interesting unit around problems of foreign money. The study of foreign exchange is especially timely, but sometimes hard to make interesting. But it took on new and unusual interest when actual money from many lands was handled.

Motivation

The teacher brought the coins to school one day—francs, centimes, piastres, and others—and told the pupils something of the difficulties of shopping in foreign places. The boys and girls were tremendously interested and asked so many questions that the teacher suggested that they might like to read some on topics related to the coins. She said that she had been trying to find time to mount the coins on cards, with brief explanations of their values and their nationalities. The pupils begged to do this for her.

Problem

They decided to find the value of each coin in relation to other coins of the same country, and its value in United States money. The coins were to be mounted upon a large card with tables of value printed beneath them.

Procedure

Committees were chosen by the class to work on the many sub-problems, such as getting the cardboard; printing the tables; classifying and arranging the coins; reading and reporting on related topics of interest; writing news items for the school paper on the progress of the unit. One artistic pupil asked for the task of sketching people of foreign lands, in costume, holding the national flags. These figures were to be placed beside the coin arrangement for the different countries.

Difficulties Overcome

It was soon found that one cardboard sheet would not be satisfactory, as it would be too crowded and inartistic. The class then decided to use a 12×18 cardboard for each country—England, France, Germany, Italy, Switzerland, Monaco, Greece, Egypt, Palestine, Syria, and Turkey.

Considerable experimentation was necessary before the coins were satisfactorily mounted. The committee in charge first tried library paste. This held well until dry, when the coins dropped off of the cardboard. But the class was delighted to see that a perfect impression remained in the dry paste left on the card. After careful consideration they departed a little from the original plan, since they had found a way to show both sides of a coin. Paste impressions, showing each side of each coin, were made for the room exhibit cards. Then the coins were mounted with glue on smaller cards for the teacher's collection. These cards were numbered with the same numbers as the larger cards and kept locked in the exhibit case.

The inscriptions on the coins, being in foreign languages, could not be read without help. The committee in charge of translation found some help in reading and some in asking people of the community. There was great pleasure in guessing the meaning of words which looked similar to our words. The teacher had brought back a booklet of instructions for tourists which had been given her on a Mediterranean boat. In this booklet was information concerning the names and values of the units of money to be used in different ports. Instructions were given in parallel

English and French columns, and pupils had great fun in believing that they were reading French.

English money presented more difficulties than others because the decimal system is not used. The mathematics book had to be consulted frequently.

When the cards were finished the class went into a committee of the whole to plan the best arrangement for exhibition in the room. The publicity in the school news sheet had interested other grades and the completed exhibit had to be explained to numerous visitors.

Related Learnings

In addition to the knowledge gained concerning foreign exchange, the pupils were richer in sympathetic experience of life in foreign lands, especially manners and customs and economic life.

Through consultation of maps and reference charts, geographical relations were better understood. Throughout the progress of the unit, much reading for background was done. Considerable dictionary work was done for flag drawing and explanation of terms used.

Some current events magazines and histories were consulted in an effort to know more about tariff regulations. Steamship and railway advertising material gave wider knowledge of travel customs and conditions.

Material which pupils found was supplemented by talks given by the teacher and study of pamphlets and postcards which she had brought home from abroad. Incidentally, the passport, an exhibit which fascinated all, became the motivation of a later unit.

Correlation with English was secured through summaries of outside reading, newspaper reports, and creative effort given to description of a day in a foreign market. Some knowledge of foreign language, very slight but very inspirational, was obtained.

Best of all, the unit showed the value of cooperative effort and the possibility of unifying school subject matter and activities for greater pleasure and understanding.

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The Future of Geometry

By MARGARET JOSEPH
Shorewood High School, Milwaukee, Wisconsin

TEACHERS of mathematics are often confronted with pupils asking, "Why should I study mathematics?" or "Where shall I ever use geometry?" Such statements made by pupils are easily answered by any mathematician; so are of little concern. The real problem lies in the criticism from adults, for many of our fellow teachers and executives believe that mathematics should have no place in the new curriculum. Just what should be taught in its place has not been determined, for their criticism is of the destructive type, not the constructive. It is only natural that every teacher should feel that his specialty is of importance, so regardless of all the unfavorable comments some people make about mathematics, I still feel that it should be studied by all pupils capable of doing high school work.

Too many adults think of high school mathematics being used only where you let x equal the unknown quantity when solving a problem. At one time one of my geometry pupils, in search of practical applications, asked a sign builder if he had occasion to apply geometry other than "propping up billboards." The man replied that he wasn't using geometry, just common sense, to prevent the wind from blowing the thing over. I am reminded of a statment made by Dean Schlicter when he said, "Go down deep enough into anything and you will find mathematics." There is much food for thought in that little statement!

Mathematics is used everywhere, not only in the more familiar applications to building from the primitive to the most modern, in bridges, in all forms of transportation, in the professions, in business, and in artistic design, but also in nature, and in everyday reasoning. How difficult it is to describe a row of hard maple trees without using geometrical terms, for they are perpendicular to the ground, parallel to each other, having branches both oblique and perpendicular, and symmetrical leaves which alternate on the twigs. Geometrical figures are found in flowers, in snowflakes, and in heavenly bodies. One does not need to be a very close observer of nature to see these and many other forms. A study of geometry makes one appreciate these forms in nature as well as the many applications of geometrical figures made by man in designing insignia, printed materials, linoleum patterns, and others too numerous to mention.

The training in logical reasoning, which teachers of geometry aim to develop in every pupil, should carry over in all other high school courses and throughout life. Training to tell why a statement is true before it may be quoted as a fact, should develop the habit of knowing an authority for whatever opinions or conclusions one might arrive at, whether it be history, mathematics, or home discussions. It eliminates the dogmatic, prejudiced ideas which gullible individuals are so prone to absorb and logical, sane reasoning is substituted in its place. Because a pupil fails to reason in other classes or after leaving high school, does not mean that reasoning taught in geometry does not function and therefore has no value. The highest type of thinking is demanded in the mathematics class, and to firmly establish the habit, the same kind of reasoning must be required in all classes. Teaching correct English used to be considered the job of the English teacher, but effective results were shown only since all teachers cooperated by insisting on correct usage in every class. The same should be true in teaching reasoning.

I have heard adults criticize mathematics saying we can get along without it. Those interested in engineering, research,

business administration etc., can be taught all the necessary mathematics in college. Such statements are no doubt familiar to you. I often wonder why mathematics and Latin should have the brunt of this criticism. We seldom hear other subjects included, yet we might go down the list of subjects offered in high school, crossing out one after the other; in fact, the greater percentage, because we can get along without them. In fact, we can do without most of them much more easily than mathematics, since our livelihood is bound to be associated with numbers. But what narrow individuals we would be! I do not want to give the impression that I see no value in the study of poetry, for example. I certainly do. My study of Shakespeare and Browning has meant a great deal to me. I wonder if those who condemn mathematics on the basis that one can get along without it ever go around quoting Chaucer, Keats, or Shelley. Using their own form of reasoning, courses in poetry should be taboo, for they are not necessary for one's existence. They might defend it for its cultural value, but those same individuals see no culture in the study of mathematics to appreciate the many scientific articles appearing in newspapers, and magazines, such as articles on the new stream line cars. Similarly, why study chemistry? If interested in being a chemist, study it in college. The same might be said of other subjects. I wouldn't care to eliminate from my understanding all work which I absolutely didn't need; it would be too much of the savage level.

Much criticism comes from those who are forced to teach mathematics when unprepared, or, if qualified, may have been asked to teach mathematics when science or history was their preference. At a recent meeting of mathematics teachers, I heard an instructor of physics say he was forced to take an algebra class now and then. One could sense his dislike of the subject the moment he spoke. He had no use for their textbook just because it used the word "transpose" instead of referring to axioms. Furthermore he would eliminate every part of the course which is difficult to teach so as to ease the work of the teacher and eliminate failure on the part of the pupils. Teaching mathematics requires a great deal of patience and an endless amount of work. The less innate ability a pupil has, the more special attention he requires. All modern textbooks have already eliminated the useless material while the more difficult exercises are assigned to the brighter pupils. I feel that further cutting of material is unwise and is unjust to the many individuals preparing for college. Differentiated assignments may be made giving to all the training in reasoning and an appreciation of geometrical forms.

Although it is far from perfect, I like the plan recently instituted in our high school not only in geometry but in all mathematics courses from the seventh through the tenth grades. Homogeneous grouping is made on the basis of mathematical ability in the previous course. Prognosis on this basis is not infallible but we have found a very high correlation between progress in algebra and that in geometry.

Although the same method of grouping is used in the junior high school, I shall refer only to the geometry classes. We use the letters A, B, C, D, and F in marking pupils where F means failure. As near as possible, pupils doing A work in algebra are assigned to one section of geometry, the B pupils another, and so on. We have five sections, the lowest being composed of the very weakest of the D pupils and usually consisting of those who need three semesters to complete their algebra. Since pupils of this type form our real problem, I want to emphasize the work of this group. These pupils have a greater dislike for the subject than most pupils, for it is only natural for us to like that work in which we excel. By making a very gradual approach to the subject, and bringing in interesting constructions and designs, much of the fear of the subject is elimi-

nated. After the work is introduced, the pupils work at their own rates, some finishing in two semesters while others must attend summer school. As soon as a pupil completes a given assignment, such as a week's work, he is given a test. If he passes, he proceeds with the next assignment sheet. If he fails, his errors are explained, further drill problems are assigned from other texts or workbooks, and after that a retest is given. This is repeated if necessary. With the allotment of more time, most of the pupils complete the course. There are seldom, if ever, pupils who reach the tenth grade who are unable to master geometry if it is taken in smaller doses and spread out over a longer period of time. They usually show a good deal of pride in knowing they are capable of doing the work, which appeases their former feelings toward the subject. It reduces failures to a minimum; what more could our critics ask? The teacher, on the other hand, maintaining the standards of her profession, has the satisfaction of giving credits to those who earned them and not doled out to pupils who put in time but not effort, protecting themselves under the alibi that the course is too difficult, nor to pupils completing a course so slashed that it would make a real mathematics teacher blush to label it geometry.

It is my opinion that our success in dealing with this group will determine to a great extent the future of geometry. With much of their fear and hatred of the subject gone, incoming classes will not come prepared to dislike it. I always feel that if I have a pupil of low ability who likes the subject, I can teach him, but when he dislikes mathematics and thinks he is going to fail because someone told him it is hard, he soon sets up a defense mechanism hard to break through.

It is very difficult to try and change an adult's set opinion about geometry so by concentrating our efforts on the pupils of today, the future will be more promising. Adults are less apt to condemn a subject they liked. A man recently told me that he never used geometry, he never would use it nor would his child. His set determination was the best proof that he wasn't using it, but he was to be pitied. No one could change his opinion of anything regardless of what proof was offered for he was so set in his thinking that prejudice ruled over reason. So let us, as I said before, concentrate our efforts on making pupils today like geometry not only by making it interesting and useful, but by adapting methods to the pupil's ability and so reduce failures and hence criticism to a minimum.

IMPORTANCE OF MATHEMATICS

THE WORLD in which we live is incurably mathematical. Every human being is born into a physical universe in which quantity, shape, and size play an indispensible rôle. The geometric principles of equality, symmetry, congruence, and similarity are implanted in the very nature of things. It is apparent, for example, that we cannot make or manufacture the simplest article without giving the constant attention to its form, its dimensions, and the proper relation of its parts. The art of measurement permeates the fabric of modern civilization at every point. It underlies all applied work in engineering, technology, and manufacturing. Without measurement and computation the world of science would cease. Algebra furnishes economical methods and formulas for many of these computations. Trigonometry, being essentially the art of indirect measurement, forms the necessary background for the making of maps and survey plans of all sorts. It underlies the art of navigation. Our entire civilization, our sciences, our modes of thinking, have a mathematical core.—WM. BETZ.

An Attempt at Solving the Problem of Individual Differences

By CLARA D. MURPHY
High School, Evanston, Illinois

FOUR YEARS of Mathematics are offered in Evanston Township High School, as follows: In the first year three courses, called Algebra 1, Mathematics 1, and Business Training; in the second year three courses—Plane Geometry, Mathematics 2, and Bookkeeping; in the third year two courses—a second year of Algebra and Mathematics 3, which consists of a semester of Algebra and a semester of Solid Geometry; and in the fourth year one course, consisting of a semester of Trigonometry and a semester of College Algebra.

Mathematics 1 is an opportunity course for pupils who do not have the ability to do the college preparatory course in Algebra 1. It gives the same credit for graduation that Algebra 1 gives, but it does not carry college entrance credit. Pupils who complete this course may do one of three things, depending upon their record: take no more mathematics, take Mathematics 2, or take the second semester of Algebra 1.

Mathematics 2 is an opportunity course for pupils whose ability and accomplishment seem to indicate that they should not take the college preparatory course in plane geometry. It gives the same credit for graduation as the plane geometry course, but does not give college entrance credit. Most of the pupils who take this course take no more mathematics.

Mathematics 3 is a course for superior pupils and carries 1½ credits toward graduation because the content of the year course in Algebra 3 is covered in the first semester, and solid geometry is covered in the second semester. I might say a few words in passing about the year course in Algebra 3 because the remainder of my remarks will be confined to the low ability classes. Mathematics is not required in the third year, but about half of the third year pupils elect it. This year we have two large sections of Mathematics 3, one section of pupils who plan to take college board examinations and eight sections of the regular year course. We have been very much pleased with the success of this year course, because instead of galloping through the various topics which have to be taught in the second course in algebra we have been able to do a very thorough job with each and to enrich the course with other interesting material. We have introduced work in vocabulary and underlying principles, in the history of mathematics, in detached coefficients, and in graphic solutions of cubics and quadratic systems, making equations of straight lines and parabolas, given the necessary points through which the curves must pass, variation as applied to problems in physics and chemistry, which are furnished by the teachers of those subjects, geometry problems relating to regular polygons and circles and extra work on exponents and logarithms. At the end of the year we have time to review the whole course, so that we feel we are sending these pupils on to college with a real mastery of elementary mathematics. The results in college seem to verify this.

To return to the problems of the first year, our procedure is as follows: Each year in May we visit all of the Evanston schools which send us pupils in September, two large intermediate schools and seven small schools. We speak to the pupils about the curriculum offerings at high school and have conferences with the teachers who are to help the pupils in their selection of curricula. We give at our own school Stanford Achievement Tests and group intelligence tests to all pupils who have not taken them in the eighth grade. In general, this is done before they are assigned to classes.

In June, we receive from these schools a personnel sheet for each pupil, upon which is recorded his previous school history, items in regard to his health, home, habits, attitudes, and definite recommendations from his last teacher as to the studies which he should pursue in high school, based upon his previous accomplishment, his interests, and his goal. We also receive an election card which the pupil has made out with the help of his teacher.

During the summer I go over these election cards, personnel sheets, and tests, with two things in mind:

1. To find pupils who have elected to take college preparatory curricula when their previous record does not warrant it;
2. To find pupils who are so weak in English or arithmetic or both that they should be put into low ability classes.

When the intelligence test, the Stanford Achievement Test, and the teacher's rating agree, there is no question as to what courses the pupils should take; but they do not agree in a number of cases. In such cases I have found that the teacher's rating is usually the best indication of where the pupil belongs, but I generally give the pupil a trial in the regular English or algebra if both the intelligence quotient and total Stanford Achievement score are higher than the teacher's rating. This method is not strictly scientific, I fear, since it rests largely on teacher judgment, but it has proved to be quite successful since I have very few shifts to make from the low ability class to regular algebra— 2 or 3 a year— and a small number only from regular algebra to Mathematics 1. It seems better to us to give as many pupils as possible a trial in Algebra 1 rather than to put pupils in Mathematics 1 who might do the algebra in spite of lack of ability because of their willingness to work. We find also that some pupils who have shown poor accomplishment in the eighth grade become more interested in the ninth grade subjects and do better work. In all cases of doubt, we intend to place the pupil in the regular course.

Last year, after the transfers were made, the number of pupils in the Mathematics 1 course was about 16% of the total number electing Algebra 1 the previous June. This year the ratio is smaller, as I shall explain later.

The pupils in Mathematics 1 may not receive a mark higher than 85. Up to this year, if they rated 85 during the first marking period, we transferred them to Algebra 1 before the two courses were so far apart that transfer was impossible, as we were using the same textbook in both courses. If they had an average of 80 for the year, they were permitted to take the second semester of Algebra 1 without any extra credit, in order to have a unit of algebra for college entrance. If they had an average of 75, they were advised to take Mathematics 2. If the final mark was 70, the lowest passing mark, they were advised to take no more mathematics.

The Mathematics 2 classes were then made up from 4 groups of pupils:

1. Those who had passed Mathematics 1 with a mark of 75 or 80;
2. Those who had passed Algebra 1 with a mark of 70;
3. Those who had passed Algebra 1 after failing some part of the course;
4. Those who had not made an average of 70 or better in at least three units of work during the first year.

About 20% of the pupils who elected plane geometry were placed in this course. Here again only pupils who averaged 80 were permitted to take the second semester of plane geometry without credit for a unit of geometry for college entrance. Mathematics 2 pupils who had an average less than 80 were advised to take no more mathematics.

Pupils who have had Mathematics 2 are not eligible to take Algebra 3, although they may have taken and passed the second semester of plane geometry, because their weakness in algebra, attitude,

etc., makes it practically impossible for them to pass that course. Neither are the pupils whose final mark in plane geometry is 70 eligible for Algebra 3. In order to take Mathematics 3, the year course which covers Algebra 3 and solid geometry, a pupil must never have failed any part of any subject, and must have a final mark of 85 in both Algebra 1 and plane geometry. The requirement for trigonometry is a general average of 80 or better, and not less than 80 in previous mathematics courses.

We have a special permit form which we use when parents insist, in spite of our advice, that pupils take courses for which we think they are not fitted, or when the room director thinks pupils should be given this opportunity because of extenuating circumstances. This form is signed by the parent, the pupil, the room director, and myself, and states that the pupil agrees to be transferred to another course if his work is not passing. We try to keep out of these low ability courses pupils who are failing but have ability and are not willing to work, and pupils who, their parents insist, must be prepared for college.

From the *administrative* point of view, this scheme of preparing Mathematics 1 pupils to take Algebra 1 second semester, if they wished, and Mathematics 2 pupils to take plane geometry second semester, worked well and caused little or no friction with parents, although the pupils got no extra credit for the second semester of these subjects. Two things, however, the small number of these pupils who took the algebra and plane geometry, and the high rate of failure in these low ability classes, raised these questions in our minds. Were we doing the best thing for the majority of the pupils in these classes? Was it best to plan these courses for the third of the group who might wish to finish Algebra 1 and plane geometry?

We had been trying to maintain real standards in these low ability classes. The material had been presented slowly, and every effort had been made to make it so interesting that the pupils would enjoy the classes and be willing to work, but they had been required to make a daily preparation of about forty-five minutes and to hand in work that was neat, accurate, and in good form. They had been required to master certain minimum essentials and to pass examinations, which in the end covered a semester's work in Algebra 1 and plane geometry.

This may explain the following data about our low ability classes during the past two years: Our failures in Mathematics 1 were about 20%; about the same per cent did not continue mathematics after passing this course; 30% took Algebra 1 second semester; and 30% took Mathematics 2. Last year the Mathematics 2 classes had 44% of their pupils from Mathematics 1 and 56% from Algebra 1. Their failure was about 22%, half from Algebra 1 and half from Mathematics 1 pupils. The year before more than 40% of those who took Mathematics 2 did not continue in mathematics after passing this course, and 28% elected to take plane geometry either first or second semester.

We had been feeling our way as to the content of these courses and the methods used. The first course was largely algebra. The first year we tried a review of fractions, percentage, and drill in number combinations, at the beginning of the year and decided that it was not successful. We found that we could interest the pupils in formulas, graphs, simple equations, and word problems, and do the arithmetic incidentally much more successfully than when we made an arithmetic unit. Last year we tried to organize the course in units based on formulas: formulas of measurement, formulas of science, formulas of business, etc., with a certain amount of arithmetic incorporated in each unit, a certain amount of material added for interest and motivation, and a mastery test over the minimum essentials of each unit. The topics that have been men-

tioned, with signed numbers, constituted the work of the first semester. The content of the second semester had to be governed by the fact that some of these pupils were to be prepared to go into the second semester of Algebra 1.

The methods used with this group were changed repeatedly as the group has varied greatly in attitude and accomplishment. The teachers felt that they must do everything possible to keep up the morale and interest of the group, and they succeeded in doing that. We found that we should devote more time to teaching reading and study habits and that perhaps the main factor in success was the cultivation of right attitudes. For this reason we came to the conclusion that less time spent on formal algebra, and more time spent on these things, would be of greater value to the majority of these pupils.

The content of the second year course has been worked out over a period of about four years. The teachers had to use the same textbook that was used in the college preparatory geometry classes and to cover a semester's work in demonstrative geometry, which meant doing some work with circles. They enriched the course with extra work in construction and design and appreciation of geometrical form in nature, architecture and art. They prefaced the formal proof with many experimental verifications of geometrical facts, but the formal geometry seemed to be too difficult for a large per cent of the class and the failure rate was much too high.

However, the five of us who had worked on these courses believed that it was worth our while to continue our efforts to solve this problem, because of the success we had had in making these pupils enjoy their mathematics and be willing to work at it. Last year we worked on plans for a somewhat different content and arrangement of material—plans for a two-year course in which we would break away from the college preparatory algebra and geometry. We wished to incorporate into the work of the two years sufficient algebra so that the pupils finishing the second year with a mark of 80 or better could take the regular plane geometry and be given two college entrance credits, one in algebra, and one in plane geometry, if they passed the latter course. We hoped it might also be possible for the best pupils in Mathematics 1 to take the second semester of Algebra 1 by doing some extra work outside of class.

We agreed upon the following objectives for such a two-year course:

1. To create interest in the mathematics of daily life.
2. To develop appreciation of the role of mathematics in modern civilization.
3. To develop appreciation of beauty of form.
4. To develop understanding of and skill in computation, measurement, solving problems, using formulas, using graphs, making simple geometric constructions.
5. To present important geometric concepts.
6. To develop space intuition.
7. To train in the ability to discover and use relationships.
8. To make every possible use of pupil activity.
9. To set up certain minimum essentials for mastery.
10. To develop right attitudes toward work.
11. To teach good habits of study.
12. To teach reading and accuracy of expression and the ability to determine after reading a problem the material given and the result to be obtained.
13. To develop a feeling for accuracy and orderliness and the value of proof over argument.

We decided that Schorling, Clark, and Rugg's "Modern Mathematics, Briefer Course" was the text that would meet our needs in both Mathematics 1 and Mathematics 2 in this transition year, and for the first time the pupils in these classes are using a different book from the

college preparatory classes in algebra and plane geometry.

Since the possibilities of transfer have been reduced greatly by the complete break from the college preparatory courses, pupils who said they were preparing for college were not placed in Mathematics 1 or Mathematics 2 although their previous record seemed to indicate that their chances of success were greater there than in the regular courses. Consequently, we have fewer pupils in these low ability classes. We have three sections of Mathematics 1, averaging about 20 each, which is about 9% of the total number of pupils electing Algebra 1, and two sections of Mathematics 2, averaging about 22 each, which is about 8% of the total number electing mathematics in the second year.

In Mathematics 1 we began the year's work with measurement of length with yard stick, meter, and cross-section paper. Such measurements as we made quickly showed the pupils the need of reviewing fractions, common and decimal. Estimating lengths and trying to compute the per cent of error brought up the usual weakness in percentage. Consequently, a large part of the work of the first five weeks was arithmetic. We were very careful to introduce this work only when it came in naturally with the new material and we were also careful not to drill on it so that it became distasteful. We have done a good bit of graph work, mostly statistical, which added interest and brought in the idea of ratio. In connection with adding line segments, we have taught the addition of like terms and evaluation of simple algebraic expressions such as $3a+2b$.

The second unit of work is concerned with angle measurement and constructions. We keep the arithmetic going with angle problems. We use the circle graphs to continue the study of percentage and to introduce the idea of rounding off large numbers. We begin to talk about directed numbers in connection with lines and angles, so that the pupils will have a gradual approach to signed numbers and we introduce a few simple equations in connection with complementary and supplementary angles. The constructions add interest and lead up to the making of simple designs, all of which furnishes meaningful activity both in class and out.

We plan to go on from this to the subject of areas and perimeters, which will introduce formulas and the ways of showing relationship by formulas, tables and graphs. Here the study of evaluation will be resumed and we can keep the arithmetic going by using common and decimal fractions in the evaluating.

The formula will lead to further work in equations and simple word problems. At this point we plan to take the time to make the necessary connection between reading and sentence structure, word problems and equations. One of our teachers has done some experimenting along this line with previous Mathematics 1 classes and found that it was very profitable. Most of these pupils are in special English classes, where work of a remedial character is going on and we plan to cooperate with the teachers of those classes.

From this point on to the end of the year the work will necessarily deal more and more with the mechanics of algebra, signed numbers, addition, subtraction and multiplication of monomials and binomials, and two cases in factoring, dividing by the common monomial factor and finding binomial factors by the cross product method. We shall teach the multiplication and factoring together as reverse operations. We plan also to do some work with literal fractions whose terms are monomials in order to round out the year's work with fractions and review the principles which govern their use. While this mechanical work is going on, we shall continue the work with constructions, formulas, graphs, and word problems.

From this sketchy outline you can surmise that much supplementary material has to be used along with the textbook.

Our procedure this year is for each one of us to make her own supplementary exercises. Then at the end of each unit of work we will discuss and combine this material and put it in a satisfactory form for all of us to use next year. We are agreed that it is better to have a textbook to use as a backbone than to use all supplementary, mimeographed material, especially with this type of pupil.

The course in Mathematics 2 begins with geometric constructions after a brief review of measurement of lengths, and angles. The pupils become familiar with all kinds of angles and are taught to make the five fundamental constructions with the greatest possible accuracy. Emphasis is placed upon learning to follow directions. The pupils are required to keep a notebook in which they put the finished product of each kind of work. The emphasis is here placed upon the beauty of neat work and arrangement of material on the page. The pupils are also given practice in making constructions on the blackboard and are made thoroughly familiar with each new geometric concept from all possible points of view. The textbook is merely a skeleton upon which the meat of the course is built. Many interesting exercises in construction and computation are added at this point.

The pupils become familiar with the kinds and construction of triangles and they work up through the equilateral triangle, the hexagon, the square, and the octagon, learning to construct them and to know their relations to the circle and to each other, to the point where they make original designs in their notebooks and also on the blackboard. Last year one of the Mathematics 2 teachers had some interesting borders made from combinations of regular polygons on the blackboards around her room. I have also seen some lovely gothic window designs that these pupils made. Surely such work will result in an appreciation of architecture and design, especially as the pupils are taught to recognize geometrical forms in nature, stained glass windows, architecture, and designs of primitive peoples.

A number of geometric facts are acquired by experiment such as: the triangle is a rigid figure; the sum of the angles is 180°; an equilateral triangle is equiangular; the theorems about congruent triangles, similar triangles, the square on the hypotenuse of a right triangle; and other facts necessary for the work in construction and design and the work in indirect measurement and simple trigonometry which is done later in the course. We hope to do some work with the simpler solids, making models and learning some of their properties.

In addition to this more or less intuitive geometry, we plan to teach more about simple algebraic fractions, the solution of easy quadratic equations by factoring and completing the square, simplifying the more common types of radicals, and solving the common types of word problems, so that the pupils will have been exposed to the topics usually covered in a year's work in algebra. We plan also to expose them to the nature of a geometric proof, letting them use the facts which they have acquired by experiment. We shall make an effort to teach them to analyze a theorem or problem so that the idea involved is expressed in terms of a figure and also in terms of material given and result to be obtained.

As I have said, these two courses are planned primarily for pupils who will not go to college, to give them mathematical ideas and appreciations which will make their lives more interesting and richer and give them a few tools for solving such problems in mathematics as may come their way. We hope that we may have helped them in their attitudes toward life and work. We are all convinced that their success or failure is largely a matter of attitude. With some of them no type of subject matter is easy enough for them until they have changed their attitude toward work. By hook or crook their interest and their willingness to work must

be aroused and be made to function. That is the real job.

We have also planned a way of escape for the pupils who are in Mathematics 1 but show signs of increasing ability or ambition. They might, by doing more work in factoring, division of polynomials and word problems, be able to take the second semester of Algebra 1 at the end of the year of Mathematics 1. Again, pupils who have completed both Mathematics 1 and Mathematics 2 might take the college preparatory course in plane geometry. If they passed this, we would give them two units in mathematics for college entrance, for their three years' work, since they would have covered the essentials of Algebra 1 in Mathematics 1 and 2 and had some review of it in plane geometry. How many such pupils there may be I cannot tell, but the feeling that they can take college preparatory work if they want it enough that they are willing to work hard, helps the atmosphere of the class and makes the parents more willing to accept the placement of their children. We have had practically no friction with parents over this arrangement. When they understand it, they seem satisfied for the most part. For those who are unwilling to accept our judgment, we place the pupil in the college preparatory class, getting from the parent and pupil a signed statement that they take the responsibility for the change.

We have an increasing number of children for whom special work will have to be provided. We feel that mathematics has something of value for these pupils and we think we shall be able to give it to them in our Mathematics 1 and Mathematics 2 courses. We are certain that we have found a set-up of courses that is satisfactorily organized and functions smoothly for pupils, parents, teachers, and administrators in our community.

HERE'S A NUT TO CRACK

MATHEMATICIANS, somewhat like unto medical doctors, oft disagree. One of their sore spots is the "curved line" theory. Ye Ed has been informed by competent authority that the question is one on which volumes could be written. The shortest distance between two fixed points is a matter that has vexed far abler heads than the one perched on the top of the spinal column of Ye Ed. So here's a little problem that involves curved lines and straight lines; it has to do with a spider's sense of distance between two points:

A spider once started to crawl
To the top of a smokestack tall.
His journey began at the base,
And upward he climbed into space.
He ascended the cylinder curving,
In a uniform pathway swerving,
Till at last its summit he found,
Traveling nine whole times around.
Then he circled the topmost rim,
While viewing the city dim—
And finding its distance to be,
"Twenty-eight long feet," said he,
"I'll now to the bottom descend,
In a path without a bend,
And make a final report,
Of the journey I made for sport."
And thus to the ground he came,
In a 90-yard path to fame.
He uttered only this worth knowing:
"How far was my journey going?"

Oh, yes, the answer will be given in the next issue of THE KALENDS.—From *The Kalends* of the Waverly Press Vol. 12, No. 12.

THE NUT IS CRACKED

IN THE KALENDS of December last appeared a mathematical problem under the caption of "Here is a Nut to Crack." Well, the nut was cracked by Mr. David Rosebrugh, the "leading slip-stick artist of the cost and rate department" of the Central Hudson Gas and Electric Corporation of Poughkeepsie, N. Y.

It appears that Mr. Rosenbrugh is a poet, as well as a mathematician, for here is his answer in verse:

In the spider's journey towards heaven
He travelled 369.27
Feet in a helical crawl
To the top of the smokestack tall.
In each revolution around,
Twenty-eight feet he progressed, I'll be bound,
In a horizontal plane.
At the same time, with might and main,
He rose 30 feet from the base.
The total distance per turn
Is the squareroot, as sure as fate,
Of the square of 30 plus the square of 28.
The sum of these two is 1684,
The square root is 41 feet plus .03 more;
9 times this hypotenuse
As near as I can count
Is 369.27 feet in amount.

From *The Kalends* of the Waverly Press. Vol. 12. No. 13.

THE ART OF TEACHING

A NEW DEPARTMENT

TEACHERS of mathematics are constantly being exhorted "to improve their teaching," "to teach the function concept wherever possible," and "to teach mathematics for the maximum amount of transfer"; but unless detailed procedures and explicit directions are given for the carrying out of such ideals in the class room they remain visions without technique and, consequently, are of little value.

Under the title "The Art of Teaching" the *Mathematics Teacher* is planning to introduce a new department that will contain *short* articles (800 words or less) dealing directly and intimately with the various problems that arise in the actual teaching of mathematics in the class room. It hopes thereby to actualize the above exhortations and ideals, and to make available for a greater circle of readers the successful experiences and innovations of a few.

The *Mathematics Teacher* believes that there is hardly a teacher who has not developed a successful method of teaching some topic in algebra, a theorem in geometry, or who is not responsible for an innovation,—be it ever so small—in some phase of class room procedure. To such teachers we appeal, asking them to write up their ideas and send them in. No one should fear that his ideas are too obvious or too simple; for every phase of the teaching process is important and deserves consideration. Often, many minor improvements have major consequences. Moreover, it is the professional duty of each one of us to share his discoveries with coworkers in his field.

The list below is merely a suggestive one for contributions:

1. Homework—collecting, examining, checking, and assigning.
2. Specific techniques found successful in algebra or in geometry.
3. How the topic of ..... was made interesting.
4. A successful lesson in .....
5. How to anticipate certain difficulties in .....
6. An interesting proof of the theorem on ..... (on the pupils' level).
7. The use of the function concept in teaching .....
8. The application of mathematical thinking to non-mathematical situations
9. The first week in algebra or geometry.
10. A new way of introducing .....

TEACHER'S TRUST

Where will they go who bend their head at work
Deep in their droning studies here for me?
Far unknown roads of earth and sky and sea
Will call them; dim and distant deaths will lurk
Afar for them; hard labours soon will irk
Their carefree shoulders. One day each will be
Mechanic, labourer in factory,
Airman or author, artisan or clerk.
Where they go, I will go; my teachings must
By Nature's law live ever in their brain;
Long decades hence when I am in the dust
My words will live and speak in them again.
No guardian on earth has such a trust
As this of mine; I dare not work in vain.

GERALD RAFTERY

Abstracts of Recent Articles on Mathematical Topics in Other Periodicals

By NATHAN LAZAR

Alexander Hamilton High School, Brooklyn, New York

Algebra

1. Drake, Richard. *A comparison of two methods of teaching high school algebra.* Journal of Educational Research. 29: 12–16. September 1935.

The author reports a study that was carried out in the University of Minnesota High School to compare the achievement of pupils under two methods of instruction—the individual method and the individualized group method. In the group taught by the individual method the students were allowed to proceed at their own rate and were encouraged to work out their own difficulties. "Each student did only as many problems as he felt to be necessary in order to understand the work. The teacher spent his time during his class period explaining errors on the tests taken the previous day, helping students who were having difficulty with new work and suggesting and explaining remedial work when necessary."

"The pupils taught by the group method used the same text and tests as the section described above. At the beginning of a sub-unit an assignment was made for the group with date set at which it was to be completed. This assignment consisted of maximum, medium and minimum levels as determined by the authors of the text in their teacher's manual. The new work was explained to the group as a whole at the beginning of a sub-unit and at such times during the assignment period as was convenient. Part of each class period was spent in explaining difficulties which seemed common to the group. The remaining time was spent in supervised study."

After pointing out the limitations and errors inherent in such an experiment the author makes the following comments:

a. "Students with superior ability attain a higher standard of achievement under the group method of instruction than do like students under the individual method.

b. "Students with low ability achieve better when taught by the group method than do similar students taught by the individual method.

c. "Both methods require about the same amount of time from the pupils in preparation outside the class.

d. "Slower progress is made by the pupils taught under the individual method than by those taught under the group method.

e. "Pupils in the section taught by the group method of instruction achieve consistently higher on tests given throughout the year as well as on tests given at the end of the year."

2. Garver, Raymond, *The solution of problems in maxima and minima by algebra.* The American Mathematical Monthly. 42: 435–37. August–September, 1935.

The author points out the wide applicability of two simple algebraic theorems to the solution of problems in maxima and minima. Though both the theorems and their applications have appeared in some of the older books nowhere is the real power of algebraic method pointed out. "Not only do they lead to the ready solution of a large proportion of the maxima and minima problems as stated in the ordinary calculus text, but they handle just as readily many problems involving general variables which the beginning calculus student can not solve."

3. Griffin, Carroll W. *Significant figures.* National Mathematics Magazine. 10: 20–24. October 1935.

After pointing out the importance of significant figures in applied mathematics, the writer proceeds to give rules for the retention and rejection of doubtful figures ("rounding off") in various arithmetical operations and in calculations with logarithms.

4. Herschfeld, Aaron. *On infinite radicals.* The American Mathematical Monthly. 42: 419–29. August–September 1935.

An exposition of the properties and conditions of convergence and divergence of infinite radicals of the type

$$\sqrt{a_1+\sqrt{a_2+\sqrt{a_3+\cdots}}}$$

or of the type

$$\cdots\sqrt{a_3+\sqrt{a_2+\sqrt{a_1}}}.$$

The writer is particularly interested in the number

$$K=\sqrt{1+\sqrt{2+\sqrt{3+\cdots}}}$$

and found its value to be $1.757933\cdots$.

He called the above number K, the Kasner number, in honor of Professor Edward Kasner,

of Columbia University, who suggested the problem to him.

A bibliography is included.

5. Hutchinson, C. A. *On Graeffe's method for the numerical solution of algebraic equations.* The American Mathematical Monthly. 42: 149–61. March 1935.

In many practical problems such as occur in aerodynamics and in electric circuit analysis it becomes necessary to solve equations of fairly high degree and find *all* the roots, including the complex ones.

Moreover, in the problems dealing with electric circuits the complex roots are of equal importance with the real ones. "The methods ordinarily considered in courses in college algebra, such as Horner's method, are inadequate for this purpose, being for practical purposes incapable of determining the complex roots at all, except in the case where only one pair is present."

In the article under review the writer gives an exposition of an old method that is not usually mentioned in the ordinary textbooks but was found very useful in practice.

6. Jackson, Nelson, A. *Vocabulary in beginning algebra.* School Science and Mathematics. 35: 690–94. October 1935.

A report of a study made to determine the degree of understanding of mathematical vocabulary that beginning students of algebra acquire in the first term. Although the writer himself feels that no general conclusions can be drawn from his study he nevertheless states some generalizations that are applicable, at least, to the group he experimented with.

a. "Textbook makers and teachers are trying to cover too much ground during the first part of the course in beginning algebra.

b. "In the usual course the pupil does not have time to assimilate the new language, and in many cases fails to realize the necessity for doing it. Oftentimes the teacher is at fault in not emphasizing the importance of vocabulary mastery.

c. "In most instances the meanings of words which have come from arithmetic seem to be fairly well understood and the words are used intelligently.
"In one of the most widely used texts of twenty-five years ago, 16 of the first 20 pages are entirely given over to the statement of definitions, laws and principles. Has the present-day teacher, in his reaction to such a presentation, gone too far in the opposite direction?"

7. Kilzer, L. R., and Thompson, Charles H. *Status of algebra and general mathematics in the ninth grade.* The School Review. 43: 446–50. June 1935.

The report of a study made on the following topics:

a. The aims and content of textbooks in algebra and the textbooks in general mathematics most commonly used in the state of Wyoming.

b. The acceptance of the two types of courses by higher institutions of learning.

The conclusions are:

a. The authors of textbooks in algebra and in general mathematics have similar aims.

b. Both types of textbooks stress the correlation of algebra with arithmetic, geometry and trigonometry.

c. The textbooks in algebra give less attention to arithmetic and geometry and more attention to algebra than do the text books in general mathematics.

d. Both types devote a large percentage of space to algebra.

e. The number of high schools in Wyoming using books in general mathematics has nearly doubled in recent years.

f. About a half of the 23 universities questioned will accept credit in general mathematics in lieu of algebra.

8. Ward, Jas. A. *Using the Hessian to solve a cubic equation.* National Mathematics Magazine. 9: 235–40. May 1935.

A description of a method of solving a cubic equation that is different from Cardan's and also much easier. Unfortunately it is impossible to incorporate this technique in the usual syllabus of mathematics in the high school or college since the obtaining of the Hessian of a function presupposes a knowledge of partial differentiation.

Arithmetic

1. Brownell, William A. and Chazal, Charlotte B. *The effects of premature drill in third grade arithmetic.* Journal of Educational Research. 29: 17–28. September 1935.

The authors concede that "drill increases, fixes, maintains and rehabilitates efficiency. The question that they are interested in is—"What contribution, if any, does drill make to raising the level of children's performance in arithmetic, to promoting growth in mature forms of quantitative thinking?" The investigation, reported in the article, leads them to answer the question in the negative. They are, however, very

careful to point out that this conclusion "in no way implies that drill has no place in arithmetic."

A bibliography of 20 items is appended.

2. Buckingham, B. R. *When to begin the teaching of arithmetic.* Childhood Education. 11: 339–43. May 1935.

After reviewing the arguments that are usually given for the postponement of the teaching of arithmetic, the author presents the evidence and the reason for the contrary point of view. He concludes that "if the time to begin is when the child is ready and has use for the subject, then we should begin the teaching of arithmetic as soon as he comes to school."

3. Davis, Mary Octavia. *Arithmetic seatwork for beginners.* The Instructor. 44: 24. September 1935.

A page of pictures, within the drawing ability of children beginning arithmetic, that will help inculcate the concept of the numbers 0 to 9.

4. Deans, Edwina. *Materials and methods.* Childhood Education. 11: 359–66. May 1935.

"This article represents an attempt to give some practical suggestions with regard to teaching numbers in the primary grades. The materials and methods to be described have proven their worth under the test of class room use." The text is profusely illustrated and contains many details of class room procedure.

5. De May. Amy J. *Arithmetic meanings.* Childhood Education. 11: 408–12. June 1935.

An exposition of the objectives that a first grade teacher must hold before her in order to render meaningful the number symbols and their uses.

6. Faubel, Ruth. *Arithmetic worked up from art.* School Arts. 35: 52. September 1935.

The writer points out many situations in which arithmetic and art can be correlated.

7. Gould, Gertrude. *Socializing arithmetic.* The Instructor. 44: 54+. October 1935.

An enumeration of situations having social value that may be utilized in the teaching of arithmetical facts, combinations and processes.

8. Grossnickle, Foster E. *To check or not to check?* The Elementary School Journal. 36: 35–39. September 1935.

The study is concerned with the advisability of using multiplication to check problems in division, when the divisor is a one-figure number. The author believes that "the data in this investigation point unerringly to the conclusion that in grade 4 checking does not result in accuracy in long division." The reader is warned "not to generalize about the value of checking in the other three processes."

9. Kelley, Anna A. *Teaching remedial arithmetic.* American School Board Journal. 91: 44+. August 1935.

An analysis of various errors that pupils make in learning arithmetic, and a description of successful methods of remedying them.

10. MacLatchy, Josephine. *Number abilities of first grade children.* Childhood Education. 11: 344–47. May 1935.

The writer is one of the authors of the well known and important study on *The number abilities of children when they enter grade one* that appeared in the Twenty-ninth Yearbook of the National Society for the Study of Education, 1930, pp. 473–524. The article under review is a clear and interesting summary of the above mentioned study.

11. Manuel, H. T. *A comparison of Spanish-speaking and English-speaking children in reading and arithmetic.* The Journal of Applied Psychology. 19: 189–202. April 1935.

An investigation of the retarding effects of a bilingual environment on accomplishment in school subjects. Among the conclusions drawn, the following is of special interest: "In both groups there was evidence of a positive relationship between socio-economic level and achievement in reading and arithmetic."

12. Masterton, James. *The school shop.* The New Era (London). 16: 183–86. July–August 1935.

An exposition of the possibilities that the school store offers for the teaching of arithmetic.

13. McLaughlin, Katherine. *Number ability of pre-school children.* Childhood Education. 11: 348–53. May 1935.

The author reports "an investigation whose purpose was:

a. To analyze in pre-school children the development of three phases of quantitative experiences, namely, counting, recognition of number aggregates, and combination of aggregates; and
b. To determine to what extent the development of ability to deal with number is conditioned by general intelligence."

14. Mossman, Edith L. *Raising the degree of mastery in seventh and eighth grade arith-*

metic. California Journal of Secondary Education. 10: 391–96. May 1935.

A description of the methods that were actually used by the members of the department of mathematics in the Garfield Junior High School, Berkeley, California, in an attempt to raise the degree of mastery and understanding of arithmetical processes.

15. Myers, Garry Cleveland. *Fractions.* The Grade Teacher. 53: 52+. September 1935.

A development of the common problems that occur in the teaching of fractions. Specific examples and details of class room procedure are given.

16. Polkinghorne. A. R. *Young children and fractions.* Childhood Education. 11: 354–58. May 1935.

A summary of the writer's unpublished master's thesis, "*The concepts of fraction of children in the primary school,*" University of Chicago.

17. Rich, Frank M. (a) *Fun with arithmetic processes.* American Childhood. 20: 13–14+. June 1935. (b) *Pupils' share in fourth- and fifth-grade arithmetic.* American Childhood. 21: 14–15. September 1925.

A description of an activity program for the teaching of arithmetical facts and processes.

18. Rosenberg, R. Robert. *Business mathematics test no. 10.* The Business Education World. 16: 73–74. September 1935.

The test is designed to test knowledge of graphs, depreciation, denominate numbers and practical measurements.

19. Sleight, E. R. *Early American arithmetics.* National Mathematics Magazine. 10: 9–12. October 1935.

An exposition of the topics in the textbooks on arithmetic that were published in the United States in the eighteenth century and in the beginning of the nineteenth.

20. Stevens, Marion Paine. *Instructional materials for teachers.* Childhood Education. 11: 371–72. May 1935.

The writer gives a list of instructional materials and supplies the names and addresses of the firms manufacturing them.

21. Stretch, Lorena B. *The value and limitations of drill in arithmetic.* Childhood Education. 11: 413–16. June 1935.

After pointing out the prevalence of the drill method of instruction the author analyzes very carefully (a) what drill does, (b) the inadequacy of drill, (c) the place of meanings in arithmetic, and (d) the value of drill.

22. Thrush, M. E. *First grade arithmetic based on Winnetka procedure.* School, (Elementary Edition, Toronto.) 24: 50–53. September 1935.

An exposition of the program followed in the schools of Winnetka, Illinois, incorporating the recommendations of the Committee of Seven, of which Carleton Washburne, the superintendent of schools in that city, was the chairman.

23. Watson, Stanley A. *Arithmetic; junior third to senior fourth. (grades 5–8).* School (Elementary Edition, Toronto.) 24: 27–33. September 1935.

The author describes the objectives to be kept in mind in the teaching of arithmetic, and outlines a practical program to realize them.

24. Wesley, Marion J. *Social arithmetic in the early grades.* Childhood Education. 11: 367–70. May 1935.

A detailed enumeration of the numerous opportunities for number experiences that arise in the life of a young child.

Calculus

1. Gibbins, N. M. *A chapter on limits.* The Mathematical Gazette. 19: 249–55. October 1935.

A rigorous exposition of the notion of limit as it occurs in (a) square root, (b) logarithms, (c) exponential functions, and (d) arctangent function.

Geometry

1. Garver, Raymond. *Bieberbach's trisection method.* Scripta Mathematica. 3: 251–55. July 1935.

The exposition of a method of trisecting an arbitrary angle which requires, in addition to the straight edge and compasses, only one construction with a movable right angle. In place of Bieberbachs's proof involving vector methods the writer gives one of his own which uses only elementary analytic geometry. Moreover he derives two other methods of trisection to which Bieberbach's method is equivalent. A bibliography is appended.

2. MacKay, Roy. *A metrical analogy between the triangle and the tetrahedron.* School Science and Mathematics. 35: 730–36. October 1935.

The writer reviews at first a few of the simple, though not generally known, properties of a tetrahedron and their analogues to the plane triangle. He then compares the metrical property of a point on the Euler line of a triangle with a similar property of a point of the

corresponding line of a tetrahedron. A bibliography is included.

3. Weisensee, J. P. *An experiment in plane geometry.* Nebraska Education Journal. 15: 282+. September 1935.

A description of an experiment at Tekamah, Nebraska. After an introductory course of two weeks the students were given workbooks in geometry to supplement their regular text, and were allowed to complete the various units of the term's work at their own rate. The advantages of such an arrangement for the slow and the bright pupils are pointed out.

Trigonometry

1. Gruenberger, Alice G. *Slants on surveying.* The Wisconsin Journal of Education. 68: 12–13. September 1935.

The writer describes the enthusiasm for mathematics aroused by measurements made with home made instruments, and includes sketches and directions for constructing them.

2. Hodge, Frederick H. *Some general formulas suggested by an elementary identity.* National Mathematics Magazine. 10: 13–17. October 1935.

Some interesting generalization arising out of an attempt to prove the identity

$$\cos 20° \cos 40° \cos 80° = \tfrac{1}{8}.$$

3. *The solution of a triangle, given three sides.* The Mathematical Gazette. 19: 180–88. July 1935.

A report of a discussion that took place at the meeting of the Mathematical Association (England) on January 8, 1935. The participants point out the advantages and disadvantages of various methods used in solving triangles, with particular reference to checking.

Miscellaneous

1. Lawther Jr., H. P. *An application of number theory to the splicing of telephone cables.* The American Mathematical Monthly. 42: 81–91. February 1935.

At a time when mathematics is being attacked, the following quotation from the article will surely be welcome. "Some time ago in connection with the placing of a long telephone cable the writer had occasion to attempt the specification of a splicing scheme. . . . The task, superficially so simple proved to be one of most intriguing difficulty, and the pursuit of the solution led a confused investigator stumbling into the province of number theory. That speculation upon an art so mundane as that of telephone splicing should have led to a proposition in the oldest and most neglected branch of mathematics seems specifically worthy of note, for few applications so practical have been found."

2. Minois, S. *L'Enseignement des mathématiques en France dans les lycées et collèges.* The Mathematical Gazette. 19: 189–91. July 1935.

A detailed exposition of the teaching of mathematics in the French colleges. The underlying educational philosophy and objectives are also pointed out.

3. Miller, G. A. *A characteristic feature of Babylonian mathematics.* School and Society. 42: 438–39. September 28, 1935.

The author notes that "the Babylonian mathematical literature furnishes the only known instance of a system of numerical notation in which exactly the same symbols are employed for fractions and integers."

4. Miller, G. A. *The backwardness of early American mathematics.* The Mathematics Student (India). 3: 8–11. March 1935.

A recitation of many interesting historical facts showing the backwardness of early American mathematics.

5. Moulton, E. J. *The training of mathematics teachers.* School and Society. 42: 195–96. August 10, 1935.

A summary of the report that appeared in the American Mathematical Monthly, in the issue for May 1935.

6. *Readings in the literature on teaching, with special reference to mathematics.* The American Mathematical Monthly. 42: 472–76. October 1935.

A recommended list of readings for prospective teachers of mathematics in the high schools and colleges. The list was drawn up in response to a request from the Commission on the Training and Utilization of Advanced Students of Mathematics.

7. Seasholes, H. C. *Mathematics and the social studies.* Ohio Schools. 13: 217+. September 1935.

The writer points out that our mathematical sequence of arithmetic, algebra and geometry, trigonometry, analytic geometry and calculus "has been built to care for the needs of science and engineering. It has functioned well and should be continued. What is now needed is a parallel mathematics curriculum to care for our needs in the social field." Such a curriculum should contain mathematics for the home, for the job, and for society in general.

8. Thomas, W. R. *John Napier*. The Mathematical Gazette. 19: 192–205. July 1935.

"The main object of this article is to suggest that John Napier took the word 'logarithm,' though not the idea, directly from the Psammites (Avenarius, Sand-Reckoner) of Archimedes."

9. Wren, F. L. *The function concept and its possibilities in the teaching of secondary mathematics*. Peabody Journal of Education. 13: 19–27. July 1935.

An exposition of the function concept, with a few examples to illustrate the range of its application.

NEWS NOTES

The Illinois Division of the National Council of Teachers of Mathematics was called to order by the chairman, Miss Martha Hilderbrandt, in room 300, Mathematics Building, University of Illinois, on November 22, 1935, at 9:00 A.M.

Miss Hilderbrandt appointed the following committee to nominate one member to replace the retiring member of the executive committee of the Illinois Division, and a delegate to the Saint Louis meeting of the National Council:

Dr. Helen Taylor, University High School, Urbana, Chairman

Professor E. H. Taylor, Charleston (Eastern Teachers College)

Professor E. W. Schrieber, Western Teachers College, Macomb

Adah Pratt, Elgin High School, presented a paper on "Teaching Pupils How to Study Algebra."

Professor Arnold Emch, University of Illinois, presented a paper on "Models of Figures in Non-Euclidean Geometry."

Mr. E. W. Schrieber, Secretary of the National Council, spoke to the group concerning membership on the National Council, which includes subscription to the *Mathematics Teacher*. He also recommended the yearbooks of the council. Mention was made concerning the meeting of the National Council at Saint Louis.

Mary A. Potter, Racine, Wisconsin, presented a paper on "The Human Side of Mathematics."

Beulah Shoesmith, Hyde Park High School, presented a paper on "The Value of a Mathematics Exhibit."

Motion made to have the secretary express the feeling and wishes of the section to Professor E. B. Lytle who was at the time ill at the Palmer Sanitorium in Springfield, Illinois.

The meeting then adjourned for the noon recess.

The meeting was called to order about 2:00 P.M. by the chairman.

Dr. Helen Taylor, chairman of the nominating committee, presented the following report:

We wish to nominate Dr. Miles Hartley, University High School, Urbana, Illinois, for the election to the executive committee to serve three years, and to act as secretary of the executive committee next year.

We also wish to nominate Mr. M. J. Murphy, Urbana High School, as a delegate to the meeting of the National Council at Saint Louis.

Moved and seconded and voted that the recommendations of the nominating committee be accepted.

Moved, seconded, and voted that the executive committee for next year shall be Mr. E. G. Hexter, Township High School, Belleville, Chairman; Professor C. N. Mills, Illinois State Normal University, Vice-Chairman; Dr. Milles Hartley, University High School, Urbana, Secretary.

Mr. E. W. Schrieber spoke concerning the meeting of the Central Association of Science and Mathematics Teachers at Chicago on November 29 and 30.

Mr. C. N. Austin, Oak Park High School, spoke concerning values received as members of the National Council.

General topic for the afternoon session: "How Can We Show the Relationship of the High School Courses in Mathematics with Each Other and with Allied Subjects?"

Miss Gertrude Hendrix, Eastern Teachers College, Charleston, presented a paper on "Algebra as a Tool for Strengthening Arithmetic."

Mrs. Elsie Parker Johnson, Oak Park and River Forest Township High School, presented a paper on "The Relation of Geometry to Allied Subjects."

Jessie D. Brakensiek, Senior High School

Quincy, presented a paper on "Third Year Mathematics, Whence and Whither?"

A discussion of these papers followed.

The meeting adjourned.

C. N. MILLS
Secretary

Professor Orata of Ohio State University has asked us to indicate the following corrections for his article on "Transfer of Training and Educational Pseudo-Science" which appeared in the May (1935) issue of *The Mathematics Teacher*.

"The last sentence on page 273 should read 'It will be noted in Table III below that, while the number of classroom investigations remained practically the same in the two periods (1890–1927 and 1927–1935), the number of laboratory investigations decreased by more than 50% in 1927–1935.'

"Table III on page 274 should read:

	1890–1927	1927–1935
Classroom.....	51	45
Laboratory.....	56	23

The change in figures does not in the least alter the conclusion."

Mathematics section meetings as a part of the programs for the Nebraska State Teachers Association were held October 24 and 25 in each of the six districts as follows.

District Number One at Lincoln:

President: R. O. Severin, Roca.

Secretary: Ann Arnold, Beatrice.

"Remedial Work in High School Mathematics," Supt. R. B. Thompson, Beaver Crossing.

"Humanized Mathematics," Miss Helen Dunlap, Lincoln High School.

District Number Two at Omaha:

President: William Sudman, Oakland, Nebraska.

"A New Method of Teaching Geometry"—Miss Marian Dodderer, Tekamah High School.

"Mathematics in Secondary Education Forty Years Ago"—Professor E. M. Stahl, Midland College.

"The Place of Mathematics in the Changing Curriculum"—Superintendent, J. A. Jimerson, Auburn Public Schools.

"Mathematics For the Pupil"—Miss Eva O'Neil, Benson High School, Omaha.

Business Meeting.

District Number Three at Norfolk:

Chairman: Jennie Walker, Norfolk.

Secretary: Maud Stewart, West Point.

"A Point of View in Teaching," Dr. Clyde M. Hill, Yale University.

Group discussion.

Friday, 10:45 A.M.

Business meeting.

"Motivation in Geometry," Mildred I. Clark, Wayne.

"Motivation in Junior High School Mathematics," Abbie Bysong, Arlington.

"Modern Trends in Geometry," a paper prepared by Ruth Calendar, York and presented by Maud Stewart, West Point.

Group discussion.

District Number Four at Hastings:

President: Eva Phalen, Kearney.

Vice-President: Wilma Wilson, North Platte.

Secretary: Sherman Mulvaney, Elm Creek.

Address, Dr. A. R. Congdon, University of Nebraska.

The Motivation of Plane Geometry, Miss Helen Exley, Gothenburg.

District Number Five at Holdrege:

Chairman: Allen Lichtenberger, Beaver City.

"The Place of Mathematics and Science in our Changing Curriculum," Dr. Willard W. Patty, University of Indiana.

District Number Six at Sidney:

President: W. L. Nicholas, Oshkosh.

Secretary: Lillian Burns, Alliance.

Business Meeting.

"Why Mathematics?" Frank Barta, Gurley.

Round Table, "Reorganization and Proper Emphasis of Mathematics in Junior High."

The above programs were assembled by Prof. A. L. Hill, Peru State Teachers College, Peru, Nebraska.

Report of Nov. 7-8-9 meeting of the Colorado Branch of National Council of Teachers of Mathematics, in Denver, Colorado.

The fall meeting of the National Council of Teachers of Mathematics opened with a luncheon served to over 100 mathematics teachers at the Denver Dry Goods Co., Thursday November 7, 1935, at 12:15 noon.

Our Thursday afternoon meeting, attended by approximately 175, enjoyed thoroughly the talk by Dr. Joyce C. Stearns, Professor of Mathematics and Science, of the University of Denver. The theme was, "The Place of Mathematics in the Changing Curriculum of the Secondary Schools." Ample time for discussion was allowed, and the meeting closed at 4:00 P.M.

On Friday afternoon our theme was the same as for Thursday, and papers were presented by the following five speakers: Chester William

Harris, English; Norris F. Bush, Chemistry; Prudence Bostwick, English; Don Harrison, Social Science; and Ella A. Lawson, Mathematics. Different viewpoints were presented by teachers from these varying fields, with respect to their ideas at to our theme.

Our meeting closed Friday afternoon at the business session, where important matters were settled. Officers were elected as follows: Miss Adalyn L. Seevers, President, for a term of three years; Beth Irwin, Brush, was continued as Vice-President for one year; and L. Denzil Keigley, Denver, was continued as Secretary-Treasurer for two more years.

Mr. Harry W. Charlesworth, Denver, our retiring president, was selected as our official delegate to the St. Louis meeting of the National Council of Teachers of Mathematics, meeting Dec. 31–Jan. 1.

L. D. KEIGLEY
Secretary

Mr. W. W. Gorsline of 5826 West Huron Street, Chicago, Ill., writes that he still has a supply of the very excellent address by Professor E. J. Moulton on "Mathematics on the Offensive" which he will mail postpaid for 20¢ per copy. This paper should be in the hands of every teacher of mathematics.

Here follows the program of the Detroit Mathematics Club for the year 1935–36:

November 14, 1935, Chadsey High School: "Demonstrative Geometry and the Nature of Proof," Harold P. Fawcett, Professor of Mathematical Instruction, Ohio State University. Tea 3:30 P.M. Address 4:30 P.M.

February 13, 1936, Southeastern High School: "Mathematics and the Non-College Preparatory Group." Symposium Chairman, Dr. Paul T. Rankin, Supervising Director of Curriculum and Research, Detroit Public Schools. Speakers: Dr. Rankin; Mr. Joseph V. McNally, Principal of Mackenzie High School; Mr. Fred J. Mulder, Denby High School; Mr. Joseph G. Wolber, Northwestern High School. Tea 3:30 P.M. Symposium 4:30 P.M.

March 21, 1936, Noonday Luncheon Meeting: "Developing the Attitude of Learning on the Part of the Pupil" Rolland R. Smith, Specialist in Mathematics in Public Schools of Springfield, Massachusetts.

Hour and Place to be Determined.

May 7, 1936, Northwestern High School, "History of Arithmetic and Algebra by Slides—5000 Years in 50 Minutes," Dr. Louis C. Karpinski, Professor of Mathematics, University of Michigan. Election of Officers. Tea 3:30 P.M. Address 4:30 P.M.

Officers: President, Miss Verna Philbrick; Vice-President, Mr. Enos H. Porter; Secretary, Miss Mildred H. Tayler; Treasurer, Mr. Wade O. Hulbert; Chairman of Publicity, Miss Hildegarde Beck.

The Association of Mathematics Teachers of New Jersey held its 55th regular meeting at Convention Hall in Atlantic City on November 9, 1935.

The following program was given:

Theme: "The New Mathematics Requirements of the College Entrance Examination Board."

"The Point of View of the Commission which Framed the Requirements," Professor Arnold Dresden, Swarthmore College, Chairman of the Commission.

"The Point of View of the College," Professor Robert M. Walter, New Jersey College for Women.

"The Point of View of the Secondary School," Mr. Rolland R. Smith, Classical High School, Springfield, Massachusetts.

Discussion: Led by Professor J. R. Kline, University of Pennsylvania, Member of the Committee on Examinations in Mathematics Alpha, Beta and Gamma of the College Entrance Examination Board.

Officers of the Society: President, Professor Albert E. Meder, New Jersey College for Women, New Brunswick, N. J.; Vice-President, Miss Marion Lukens, High School, Camden, N. J.; Secretary-Treasurer, Andrew S. Hegeman, Central High School, Newark, N. J.; Council Members: The Officers and Dean Luther P. Eisenhart, Professor Charles O. Gunther, Professor Richard Morris, Professor Virgil S. Mallory, Harrison E. Webb, Miss Amanda Loughren, Ferdinand Kertes, Miss Catherine B. Read.

The Association of Teachers of Mathematics of the Middle States met at Atlantic City, N. J. on Sat. Nov. 30, 1935. The officers and the program follow:

President: Cecil A. Ewing, Tome School, Port Deposit, Md.

Secretary: Ruth Wyatt, Woodrow Wilson Junior High School, Philadelphia.

9:30 A.M. Meeting of the Council—Green Room, Meeting Room Floor.

10:30 A.M. Meeting of the Association—Room 1344, Haddon Hall, Meeting Room Floor.

Business Meeting. Reports. Election of Officers.

Address, "Evolution of Mathematics," F. D. Murnaghan, Professor of Applied Mathematics, Johns Hopkins University.

Address, "Famous Problems of Elementary Mathematics," H. W. Brinkman, Professor of Mathematics, Swarthmore College.

Topic for General Discussion, "The Mathematics Curriculum and the New Type of Examinations

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The International Congress of Mathematicians will be held in Oslo from July 13–18, 1936. An interesting program is being prepared. Besides the scientific part, the programme will include social functions and excursions. Information may be had by writing to the Secretary, The International Congress of Mathematicians, Universitetet, Blindern, Oslo.

The following program was given at the 19th summer meeting of the Mathematical Association of America at Ann Arbor, Mich. from Sept. 9–10, 1935:

Monday Afternoon, September 9, First Session of the Association, Room 100, Hutchins Hall.

1. "The Confusion in Secondary Mathematics and Its Influence in the Teaching of Mathematics" by Professor K. P. Williams, Indiana University.

2. "Geometric Constructions Without the Classical Restriction to Ruler and Compasses" by Professor W. H. Bussey, University of Minnesota.

3. "On an Introduction to Higher Geometry" by Professor Tibor Radó, Ohio State University.

Tuesday Morning, September 10: Second Session of the Association, Room 100, Hutchins Hall.

1. "Some Recent Applications of the Theory of Elasticity" by Professor H. W. March, University of Wisconsin.

2. "Operational Calculus, Its Applications and Foundations" by Doctor Hillel Poritsky, The General Electric Company.

3. "Some Unsolved Problems of Topology" by Professor R. L. Wilder, University of Michigan.

Tuesday Evening, September 10:

Third Session of the Association, Room 110, General Library.

"Early Mathematical Books in the Library of the University of Michigan" by Professor L. C. Karpinski, University of Michigan.

This lecture was illustrated by books selected from the University Library. In addition the display cases of the General Library and of the Clements Memorial Library contained exhibits of Early American and college mathematical books.

The second meeting of the Men's Mathematics Club of Chicago was held at the Central Y.M.C.A. on Nov. 15, 1935. The following program was given:

"Surveying for the High School," by Professor A. S. Hathaway, Department of Civil Engineering, Northwestern University.

Unsolved Problems: (1) In Number Theory, by Professor H. A. Simmons; (2) In Analysis Situs, by Professor Norman E. Rutt; (3) In Mathematics in General, by Professor D. R. Curtiss.

The speakers on Unsolved Problems are members of the Mathematics Department of Northwestern University.

J. R. McDonald, *Secretary*

The first meeting of the Men's Mathematics Club of Chicago was held at the Central Y.M.C.A. on Oct. 18, 1935. The following program was given:

"High School Mathematics for the Student Who Does not Plan to Go to College," by Jos. A. Nyberg, Hyde Park High School.

"Secondary Education for the Crowd with Particular Reference to Mathematics," by Dr. Samuel Everett, University of Illinois.

A survey to list all motion pictures which have an educational value is being conducted jointly by the U. S. Office of Education (Dr. John W. Studebaker, Commissioner) and the American Council on Education in Washington. This includes not only the strict classroom film, but subjects useful to medical students, scientific workers, vocational classes, C.C.C. camps, teachers and other specialized educational groups. The survey is being made under a grant from the General Education Board (Rockefeller) and is part of the work being carried on by the American Council on Education in connection with its sponsorship of the proposed American Educational Film Institute.

More than 10,000 film catalog cards have been mailed to 1800 sources of films in this country. This card covers nearly 100 items which will result in accurate information being filed in one central office covering information necessary to judge the adaptability of the film to specific educational needs. Supplemental analyses and listings will be prepared and publicized in an appropriate manner.

Both agencies cooperating in this survey desire that this central information file be made as complete as possible. Any person or organization that has produced, now owns or has the exclusive distribution rights to any motion picture that should be included in this list, and who has not received the film catalog cards sent out under this survey, will be sent a supply of the cards upon writing to the American Council on Education at 744 Jackson Place, Washington, D. C.

NEW BOOKS

First Course in Algebra. By C. N. Stokes and Vera Sanford. Henry Holt and Company, 1935, IV+433. Price, $1.28.

This book is dominated by two purposes which are kept in mind throughout by the authors. They have selected illustrative material which would interest the pupils when a new topic is taken up and they have arranged the drill and test material so as to facilitate the teaching and learning of the subject.

The chapters are organized into learning units and each chapter contains several subunits which are intended to bring out some particular phase of the larger unit. Each subunit ends with a test which is intended to disclose how well the pupil has understood what he has studied. Practice exercises are interspersed throughout the book which can be used either as teaching or testing devices as the teacher see fit.

The book is organized so that it can be used for group teaching, for the unit method of instruction, or for taking care of individual differences.

The training and experience of the authors and their standing in the field should help to put this book before teachers of mathematics all over the country.

Elementary College Algebra. By H. W. Kuhn and J. H. Weaver. The Macmillan Company, 1935, XVI+359. Price, $2.

This book contains all the material that should be necessary for a review of high school algebra; it is written so as to meet the needs of individual differences among the students, and is organized so as to present material that is suited to the maturity of the students who take the course in college algebra.

Some chapters are so constituted as to be independent of certain others. This is done so as to make flexible programs possible. A course for students of Commerce and Administration would then be somewhat different from one for engineering students.

This is an interesting book which teachers of college algebra should examine.

Mathematics and the Question of Cosmic Mind With Other Essays. By Cassius J. Keyser. Scripta Mathematica, 1935, III+121. Price, $0.75.

This is the second of a series of mathematical books for non-mathematicians published by Scripta Mathematica. The first was *Poetry of Mathematics and Other Essays* by David Eugene Smith.

The high place that Dr. Keyser holds in the esteem of teachers of mathematics is due not only to his success as a teacher but also to his writings. He therefore needs no one to praise the books that he writes.

Neither broad mathematical training and knowledge is required for understanding the essays. All that is necessary is that the reader have "a little logical acumen," a reasonable facility in reading such discourse and be attentive to what is said.

This book is an excellent addition to the *Scripta Mathematica Library.*

Introductory College Mathematics. By W. E. Milne and D. R. Davis. Ginn and Company, 1935. XIII+383. Price, $3.

This book is intended to serve the needs of three groups of students: (1) those who study only one year of mathematics in college; (2) those who take the course as a special service course for the physical and national sciences, engineering, statistics and the like; and (3) those who are majoring in mathematics either because they intend to teach the subject or later expect to specialize in the field.

The book is really correlated mathematics on the college level consisting of algebra trigonometry, analytic geometry, and the elements of the calculus. In addition to correlating these subjects the authors have tried to bring out many applications of mathematics in the other fields.

College teachers who are looking for a book of the correlated type will do well to examine this text.

Jones-Wheat Arithmetics. By R. L. Jones and Harry G. Wheat. D. C. Heath and Company, 1935. Book One, Grades 3–4. 348 pp. Price $.64. Book Two, Grades 5–6, 372 pp. Price $.64. Book Three, Grades 7–8, 415 pp. Price $.72.

While this series offers no radical departures from existing practices, it stands out from other texts in organizing the procedures of arithmetic around a few central closely related ideas, in trying to make the pupil *understand* what he is about, in teaching him to make practical application of each newly acquired skill. The material is carefully graded.

Throughout the series the *development of general ideas* is stressed. In Book One the most important general idea is that tens, (hundreds, thousands, etc.) may be treated like ones.

Book Two emphasizes (1) the *size of parts*, which is repeatedly stressed, so that the pupil

learns to keep *sizes* clear through his understanding of the situation rather than through the guidance of a rule-of-thumb procedure; and (2) *distinctions* between the three kinds of problems—fractions, decimals, and percentage—with much practice in making the distinctions. There is constant illustration in practical situations.

Book Three deals with two important general ideas—those of *distinctions* and of *relaitons between* the three kinds of problems in fractions, decimals, and percentage. In making and applying these distinctions, the pupil is introduced to the most important personal, business, and measurement situations that are met in everyday life, and is instructed in the successful methods of attacking and handling such situations.

Drill has been distinguished from instruction. The purpose is not alone to put drill into its proper place, but to give drill its chance to be most effective. Thus, meaning rather than mere quantity of repetitions serves as the effective device for the focalization of attention.

The books are organized by units, so that the pupil studies one large topic each month. He is made conscious of that large topic through a continuous presentation of it from a variety of angles.

The books are attractively bound in bright colors and are profusely illustrated, the first book in color.

A Mathematician Explains. By Mayme I. Logsdon. University of Chicago Press, 1935, VII+075. Price, $2.

This very interesting new book is not intended to replace any of the texts of the standard courses in college mathematics, but its sponsors believe that it will be useful in the following ways: (1) to serve as a text for a one-hour or a two-hour orientation course in college, junior college, or senior high school; (2) to serve as a reading reference for first-year and second-year mathematical courses in college or junior college; (3) to service as a supplementary text for courses in the teaching of mathematics in normal schools and teachers colleges; (4) to serve as an eye-opener for the adult who knows no mathematics beyond elementary algebra and geometry but who has a healthy curiosity in the subject.

This book has provided the mathematics for a general physical science course at the University of Chicago and constitutes a method of approach that should be of interest to all teachers of secondary and college mathematics.

New Members. By F. Emerson Andrews. Harcourt, Brace and Company, 1935, 168 pages. Price, $2.

The thesis of this book was first presented by the author, in much abbreviated form in an article entitled "An Excursion in Numbers" which appeared in the *Atlantic Monthly* for October, 1934. A further section appeared in the same magazine in February, 1935, under the title "Revolving Numbers."

What Mr. Andrews has attempted to do here is to present in simple and exciting terms an idea on which mathematicians and philosophers have long agreed. The interest that was created by Mr. Andrews in his magazine articles will be further stimulated by this book.

First Course in Algebra. By Harry C. Barber and Elsie Parker Johnson. Houghton Mifflin Company, 1935, VI+425. Price $1.24.

This new book by two well known class room teachers of mathematics will be of interest to all teachers of algebra who wish to teach algebra as a symbolism for thinking. It is the purpose of this book to introduce the pupil to this symbolism, to teach him how to use it and at the same time to help the teacher in making the right distribution of the time demanded by understanding and skill.

Several devices appear in the text: (1) Class discussions intended to lead the pupil on; (2) many easy exercises both oral and written; (3) new-type tests many of which are diagnostic in nature; and many other interesting features which will appeal to all teachers who are interested in improving algebraic instruction e.g., a chapter on "Approximate Computation" is included.

The Calculus. By Hans H. Dalaker and H. E. Hartig. McGraw-Hill Book Company, 1935, VIII+276. Price $2.25.

This is the third edition of this book. No radical changes from the preceding editions have been made in this new edition but certain exercises have been replaced by new ones for the purpose of better grading and more than 200 exercises have been added. Besides these changes other improvements have been made for the purpose of making the book a more serviceable text.

College teachers of the Calculus who have used the older editions will be interested in this new book as will other teachers who are on the lookout for a suitable Calculus text.